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Mathematical Education in War-Time

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FOR a number of years past, conventional academic subjects have been subjected to critical analysis both by general educators, and by subject matter specialists. Mathematics has received a great deal of such criticism. The more radical of its critics have denied the usefulness of some of its subject matter either in the whole or in part, and have proposed extreme changes in the school curriculum. These critics have been quite generally successful in having mathematical training removed as a required part of the educational program both in the college and the high school. The more friendly critics of mathematics have admitted that some of of its subject matter needs revision; that much of its teaching has been poor (though probably no worse than in other subject matter fields); that, in view of the fact that high school education now includes nearly the entire population of high school age, the required mathematics should be selected to fit the educational abilities of the pupils; that, in view of the fact that eighth grade pupils pass into high school without having achieved desired proficiency in arithmetic, courses must be provided to correct deficiencies in early education; and that emphasis must be given to such topics as scale drawing and graphical solutions of problems, which heretofore have not received adequate attention. This is certain: If an immediate program for war service, or a future sane program in general mathematical education is to be formulated, the mathematics teachers must do the job themselves and they must also force the initiation of the program.

Let us briefly review the status of mathematical education before the advent of this war. For the past twenty years several conflicting educational groups have been subjecting the entire curriculum to drastic changes. In this struggle, little by little, mathematics, once the pre-eminent subject of the curriculum, has been relegated to a minor and unimportant place in the curriculum of the high school. In many states of the union boys can graduate from high school without having studied one single hour of any mathematics. And many that do graduate with some mathematics have had a socialized or general course for only one year of study. In view of the fact that high school education has come to include the entire population of high school age, we readily admit that an adjustment must be made. But the guidance of our schools has been such that capable students have been advised against mathematics, or at the most

advised to take only two years of mathematics. As a result, colleges were forced to lower their admission requirements from four years to three and finally to two years of high school mathematics. In many places, particularly in the West, one year and even no requirements in mathematics are now established conditions for entrance.

There have been several outstanding theories contributing to this calamitous situation. One of these is *Delayed Arithmetic*. The proponents of this theory would delay teaching formal arithmetic until pupils are more mature. Surely certain of the more difficult aspects of problems and solutions have been taught too early to many pupils; but to defer arithmetic to the fifth, sixth, or even seventh grade because it can be taught more easily there is begging the question. The arguments used for this could be used for the deferred teaching of all subjects. This same philosophy of delay has been suggested and in some cases put into practice in the high school, where the study of formal mathematics is deferred until the tenth year. This practice must be sharply criticized.

Incidental Learning in the teaching of arithmetic and higher mathematics, where a pupil would only learn facts as he met them in social situations, is an equally vicious principle when made a basic philosophy of education. In the high school the principle has its counterpart where an integrated program built around social situations would only teach the mathematics which is met in these situations.

Practical Courses are to be equally deplored. Such courses as shop mathematics, taught away from machines and tools to which they apply, mathematics for nurses, mathematics for home economics, mathematics for aviation trades, and mathematics for girls, fall in the same category. The implication is that the mathematics we teach in our regular courses is not functional, that it is so poorly taught that a shop course is necessary to show a boy how to apply common and decimal fractions to the selection of machine drills. It assumes that an entirely different course is necessary to teach a girl that if she cuts $\frac{3}{8}$ yd. of ribbon from $\frac{1}{2}$ yd. of ribbon, she will have $\frac{1}{8}$ yd. left.

The foundations of fundamental mathematics should be so well taught that the boy in the shop can apply them in the shop to the problems he meets there requiring the use of decimals, of angles and their sines and tangents, of geometric constructions, of ratios in gears and pulleys, and the making and reading of scale drawings with its easy extension to blue print reading. The fundamentals of mathematics should be so well taught that the girl in the sewing room, kitchen, or factory will be able to readily apply fractions and simple formulas to the problems she meets.

Another movement that has weakened the mathematics program is the introduction of an excessive amount of *Reasoning in Life Situations* into the subject of geometry. In many cases this has resulted in befuddled thinking and a lack of knowledge of plane geometry. Logic is not the sole right of mathematics; it should be the province of all high school subjects. The primary purposes of geometry are to teach the facts of space and the nature of mathematical thought.

Other educational theories which have contributed to the deterioration of mathematical education could be cited. But we as teachers of mathematics are in part to blame. Too frequently we have insisted that there is only one kind of mathematics, pure mathematics, and either a boy can learn it or he can not. We have forced many boys of average intelligence to be deprived of some mathematical education by such action. General mathematics has been one answer to this erroneous concept. It has taken the war however to show us how far short we have fallen in preparing boys for the mechanized, motorized, and scientific world in which we live. If we have not only the status of mathematical education at heart, but more essentially

the true welfare of our children, and above all, the safety and preservation of our nation, we must take steps immediately to correct these erroneous trends in education.

One thing is evident. Many applications of mathematics are so technical and involved that to bring them into the realm of high school mathematics is prohibitive. Nevertheless, the fundamental courses in mathematics should certainly include, as motivation, those applications which are not too technical, both for boys and girls and provide many practical uses of mathematics. More important, the boy who is going on to study navigation, meterology, mapping, artillery fire, or other branches of the service, should be so well grounded in fundamental mathematics that he can immediately apply it as a ready tool in any of these branches.

In order to arrive at some basis for the immediate type of training we should give, let us examine some of the criticisms, suggestions, and requests made by the army, navy, and industry. First, we would call your attention to the widely publicized letter of Admiral Nimitz which appeared in THE MATHEMATICS TEACHER of February, 1942. We wish to stress three startling facts in it.

1. A large per cent (62%) of the college freshmen applying for admission to the Navy failed to pass tests in arithmetic.
2. Only 23% of these freshmen had more than one and one-half years of mathematics in the high school and only 10% had studied trigonometry.
3. 75% of the failures in the study of navigation at the Navy Training School must be attributed to the lack of adequate knowledge of mathematics.

Notice that this letter does not ask the high school teachers to inaugurate special courses in navigation, avigation, and aeronautics, but requests that they make certain that *every boy* in high school takes mathematics of some type, and that every capable boy takes as much algebra, geometry, and trigonometry as his intellect permits; that mathematics be so well taught that it will function and can be used by the boy in special technical courses. These special technical courses will be provided by the government and taught by *specialists* qualified to teach them.

Second, we call your attention to a more recent letter from Randall Jacobs, Rear Admiral of the U. S. Navy and Chief of the Bureau of Navigation. We quote from this letter:

> . . . One of the greatest obstacles which the navy has encountered in its program to secure and train men for service in the navy has been the fact that high school and college applicants have not had the necessary instruction in certain essentials to permit them to be successfully trained in more technical fields. Lack of knowledge in the above essentials by trainees has hampered the navy training program tremendously. . . .
>
> The majority of secondary school graduates enter the colleges and universities of this country with only one and one-half years preparation in mathematics and the sciences and great numbers do not even have this fundamental background.

A suggested outline of study is appended to this letter in which is included plane trigonometry, solid geometry, quadratics, physics, shop mathematics and navigation, with the note:

> At least two and one half years of mathematics to include algebra, plane and solid geometry, and trigonometry should be given to all male students.

Again you will note that algebra, plane and solid geometry, and trigonometry are stressed as the fundamentals of mathematical training. Shop mathematics and navigation were also included but as we shall show later these topics can be included in the regular mathematics courses as motivation.

Third, we would direct your attention to the circular issued by the Navy, calling attention to the various types of service in which men are to be trained in the Navy and giving the fundamental prerequisite

school training to qualify for such service. There are 26 services listed; all of them call for the prerequisite of mathematics of some type of another; all call for the use of complete arithmetic and elementary algebra. Of special importance is the fact that 23 services call for a prerequisite in plane geometry, 21 in trigonometry, 7 in solid geometry, and 2 in spherical trigonometry. Notice that there is no prerequisite in avigation, navigation, or aeronautics. The navy will take care of instruction in these latter branches.

Fourth, our efforts in immediate action can be guided by a release from the War Department, Headquarters Air Flying, Training Command, Washington, D. C., on the subject of academic pretraining for aviation cadets. This release is reprinted in the *American Mathematical Monthly* for April, 1942. It is a guide for a one or two year course for those present juniors and seniors in high school and college freshmen who have not had the regular mathematics. The mathematics of 6 semester units includes algebra (25 hours), plane trigonometry and logarithms, (40 hours), solid geometry (25 hours) and spherical trigonometry (10 hours). The emphasis in such a course can be gained by such quotations as:

> . . . primarily a course in the numerical aspects of trigonometry with only the amount of analytical trigonometry which is essential for the major purpose of the course . . . ; . . . substantial emphasis on slide-rule computation with each student possessing a cheap slide rule; stress on applications of all sorts, particularly those involving vector forces, velocities, and army and navy terminology. . . . Proofs should be held to a bare minimum; great emphasis should be placed on drawing of figures and the making of simple paper models for three dimensional situations.

You will notice the stress on the practical numerical side of mathematics. You will also note that algebra, geometry, and trigonometry are basic, and should be mastered as pretraining.

Fifth, we cite an appeal published in the *British Mathematical Gazette* of January, 1941. The article calls for volunteers to teach mathematics to "prospective members of air-craft crews." The mathematics to be taught is listed under eight topics, Note that all the topics are fundamental. elementary, non-technical, and straight mathematics.

1. Common fractions, problems.
2. Converting metric units into British units.
3. Averages; ratio and proportion; percentages.
4. The Pythagorean theorem; square root. (no proofs)
5. Transposition in equations.
6. Problems using simple equations.
7. Graphs, non-algebraic.
8. Triangles of velocities.

Sixth, we refer to an address made by Major L. Barrett, Ordnance Department, Second Corps Area, to Section I of the N. J. Association of Mathematics Teachers in December, 1941, on the subject "Mathematical Training as it Affects the War Program." A statement from this authority bears repetition. He said:

> A knowledge of mathematics is one of the foundations on which the operations of modern armies are based. It cannot take the place of study of tactics and strategy, but now that armies have been so thoroughly mechanized, it is impossible to visualize their successful operation unless the officers and men of which they are composed have a thorough groundwork in mathematical training.

Major Barret listed the elementary mathematics by topics, in order of importance, that must be mastered by the future selectees. They are:

1. A knowledge of trigonometric functions.
2. A knowledge of trigonometric tables.
3. The solution of the right triangle.
4. The solution of the scalene triangle.
5. The knowledge of how to construct a curve $y=f(x)$.
6. A knowledge of how to read curves.
7. A knowledge of graphic solutions. Intersection of loci.

8. A knowledge of reading verniers, slide rules.
9. The mil system of circular measure.
10. A knowledge of logarithms.

Seventh, in lighter vein, we quote from an editorial in *The New York Times*, It reads:

> So proficient are the Red battery commanders that they often hit the target without the gradual approach of range firing, and some of them are said to be such mathematical wizards that they calculate trajectories in their heads, without the use of rigid firing tables. Germans have long talked of the Russians' "Long-bearded-professor batteries."

While the authenticity of such firing may be in doubt, it points to the value of a long ago discarded process, that of mental arithmetic. It has been said that there is only 9 seconds elapsed time between sighting an attacking bomber and the release of the bomb. Surely speedy mental arithmetic is needed in antiaircraft fire. Similarly the artillery mil as an angular unit finds its greatest favor in that it is conducive to simple mental arithmetic with results within the limit of possible error. Likewise the Civil Aeronautics Authority Bulletin, No. 24, points out that a pilot who is his own navigator must be able to solve many problems without the use of pencil and paper. Thus he must be able to determine the approximate number of miles flown in a given number of minutes at a ground speed of so many miles per hour, as well as to estimate angles of drift and correction angles.

The quotations given thus far indicate the need for functional mathematics, so fully a part of the individual, that he can use it with skill and confidence. Several more quotations will show that the high school must be concerned with producing men with the necessary prerequisites cited above.

Eighth, in the March, 1942, issue of the *Journal of the National Educational Association*, we read:

> Since success in modern war requires complete and efficient use of the nation's total man power and since much of the effectiveness of the Selective Service System is determined by the decisions and the choices made by young men before they reach the age of twenty, the Commission recommends that educational agencies make a special effort to render advisory service to all young people, in school and out of school alike.
>
> In counseling youth with reference to continuing their education efforts should be made to conserve superior intellect and other abilities for national service in scientific research and leadership.
>
> Let our profession but answer boldly the call of the crisis and we shall fashion even out of the hard necessities of war, a school system more fit for the education of free men.

Ninth, the American Association for the Advancement of Science in their March, 1942, *Bulletin* give an indication of the tremendous job we are now facing. This article points out that for the air service alone, 150,000 men will be required this year and 300,000 more in 1943. The universities and colleges will be asked to supply half of these men or 225,000. For the necessary training, which could have been started in high school, this will mean 2,500 classes of thirty men each, a task that will strain the capacities of the colleges. About 300,000 men enter college each year in the United States. Half of these are ruled out of service by physical and other defects. Of the 150,000 left, a considerable number must be saved for scientific work and other services than aviation. The high school must step in and fill the breach by teaching the necessary prerequisite mathematics and physics to every boy capable of learning it.

Tenth, we call to your attention an article by E. U. Condon, Director (Associate), Westinghouse Research Laboratories, in the May issue of *Scientific American,* entitled, "We Need More Physicists." Dr. Condon says:

> Industry and science are being linked together by this war more closely than ever before During the present war new advances in applied physics are being made which have brought about an urgent demand for young men with sound scientific training. Any boy who has a real aptitude for scientific studies cannot serve his country better than by devoting his best efforts to learning as thoroughly as he can and as quickly as he can about mathematics, physics

and chemistry. He should not scorn the "Theoretical" fundamentals for hastily gained superficial "Practical" knowledge.

It is a great misfortune that many of our administrative officers of our public high schools have failed to appreciate the importance of sound training in fundamental mathematics and science. The quality of mathematical instruction in the high schools has sagged steadily in the past twenty years to such an extent that even the boy who gets an "A" grade in mathematics has a poor foundation on which to build his future work in science. No doubt the war will focus attention on this sorry state of affairs and reforms may result, but, in the meantime, boys who really want to help in this war should set themselves higher standards of mathematical attainment than most teachers demand.

Not only can the boy who has real aptitude for science find an opportunity to serve in the war, but with the steady advance in science-mindedness in all industry, it is safe to assume that the opportunities for scientific research in post-war industry will be much greater than ever before.

Last, a final word on this subject from the mathematics department of the United States Military Academy at West Point. In a specially prepared brochure on types of military problems involving only the use of elementary mathematics, prepared for high school teachers of mathematics, and reprinted by the Institute of Military Studies, University of Chicago, is the following significant statement:

> It is believed that effort will be more profitable if devoted to emphasizing the value of existing mathematics courses, rather than modifying them. All elementary mathematics is useful in the military services; it is desirable that this fact be widely known, so that the maximum number of students of proper aptitudes will take advantage of mathematics courses already offered. The importance of thoroughness and accuracy . . . should be stressed.

A brief comment about teaching aeronautics in the high school. No single authoritative war agency has requested it. Students need good solid mathematics and physics as a prerequisite to such study. The establishment of special courses in aeronautics in so many high schools that has recently taken place it appears to us is an excellent example of misdirected educational effort. We are specialists in mathematics and can teach it with confidence and surety. The army, navy, and technical schools have authorities to properly teach aeronautics if we supply them with students sufficiently grounded in mathematics and physics.

I disagree!

What then, as high school teachers, can we do? The teacher of mathematics anxious to serve his country and to prepare youth for positions in industry or war service, must realize first, that preparation in mathematics ranges from simple arithmetic to abstract theories of functions. It begins with a boy whose abilities in higher mathematics are non-existent, but who none the less must have thorough courses in general mathematics. He must have a confident use of simple arithmetic including common fractions, ratios, and decimals, of scale drawing, protractors, sine, cosine, and tangent functions in the solution of right triangles, and an ability to read graphs and tables.

The range of uses of mathematics spreads from this boy whose abilities are limited to general mathematics to the boy who can take all the mathematics offered him in high school, to the end that he may, after further work in college, become an officer in the army with an engineer's training, an officer in the navy, a meterologist, statistician, economist, or navigator.

A number of fundamental principles can be laid down.

1. In the war program, in this motorized age, every high school boy must study mathematics, either general mathematics or the conventional courses in college preparatory mathematics.

2. Every boy in the high school must take as much mathematics as he is capable of assimilating.

3. The mathematics the boy takes must be completely functional. That is the boy must so understand it that he can apply it with confidence and skill. It must be part of his nature, as natural in its use as his everyday speech. This definitely means better and more practical teaching so that mathematics will cease to be mechanical, meaningless drill.

4. The emergency does not mean that

boys should be urged to take courses in mathematics that they are not capable of understanding, but it does mean that more attention must be given to a guidance program which directs boys into courses in mathematics which they can take with success.

Too frequently in our high schools the adviser is a social problems man who is ignorant of the needs of mathematics in this emergency. He frequently over-exaggerates the difficulties of mathematics and many boys capable of doing mathematics well are driven away from it by unintelligent, biased advice. An adviser should be a general educator who is acquainted with various subject matter fields and appreciative of the needs for mathematically trained boys. This is true as much for the welfare of the boy as for patriotic reasons.

5. The emergency demands a greater effort on the part of all teachers of mathematics to so improve their teaching that:

a. Pupils will understand every part of the mathematics taught. This means that algebra must be taught as a reasonable development of relations between numbers and variables; that geometry is postulational thinking applied to the discovery of relations about the space we live in.

b. The more abstract materials can be given less stress at present and such materials selected as to make our mathematics functional and practical.

c. Motivation must be provided through the war effort. This means that mathematics teachers have at least a fundamental understanding of elementary problems in avigation, navigation, artillery fire, and the uses of mathematics in shop work.

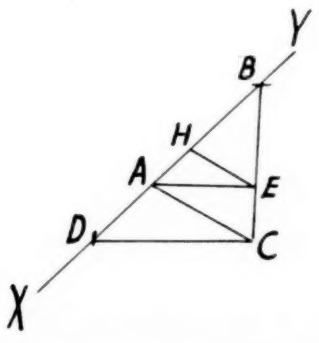
6. The emergency does not demand the introduction of special courses in high school or of innovations which would not be good mathematics under normal circumstances. It does demand that all capable students study four years of high school mathematics including trigonometry and solid geometry; that other less capable students take at least two years, and preferably three or four years of general mathematics which emphasize arithmetic, scale drawing, triangle of velocities, the ability to read graphs, charts, and tables, and the use of numerical trigonometry.

Several times there has been mentioned the advisability of providing material for motivation in the regular courses in mathematics related to the war program. The brochure reprinted by the Institute of Military Affairs, University of Chicago, contains many such examples. The alert teacher can find such examples with a little study of the existing literature. We shall illustrate what we mean by a simple example on radius of action. In the ninth grade, when studying distance-rate-time problems, this would offer excellent motivation. [*If the out-going speed of an airplane is O and its incoming speed is I, how far can the plane go before turning back to the same base if its gasoline supply will last h hours*?]

This simple problem can be solved by the formula

$$R=\frac{h(IO)}{I+O}$$

where R represents the radius of action. In actual flying the problem is solved by scale drawing as follows:



From A, the starting point the velocity is laid off as AC. The track to be patrolled is drawn through A as XAY. From C, with a

radius equal to the air speed, arcs are described cutting the track at B and D. Through A draw AE parallel to DC, and at E draw EH parallel to AC. Then AH is the radius of action for 1 hr. AB the going out speed O, and AD, the incoming speed I. Thus in the above figure

$$AH = \frac{AD \cdot AB}{AD + AB}.$$

This relation might well occur in any plane geometry text as an original exercise under the topic of similar figures. The plane geometry student will certainly appreciate the value of geometry when he is shown this war- and peace-time application.

To summarize: The army, navy, industry, and scientific organizations indicate that the mathematics teachers can be of immediate aid in the war emergency by securing the following:

1. Every boy in high school should study mathematics according to his abilities, the more capable taking four years of mathematics including trigonometry and solid geometry.
2. Advisers must realize the acute need for mathematics and direct all capable boys into such courses.
3. It is not necessary to create new courses or to entirely reorganize the present courses in the war-time effort. If our objectives are valid, and our methods of teaching will obtain these objectives, all that is needed is an added emphasis on the practical.
4. Motivation can be provided through the use of simple formulas, scale drawing, and geometric theorems directly involved in military tactics. The teacher should know these applications.
5. Good teaching of straight mathematics by fully qualified teachers is what is needed. Such teaching is a far superior procedure than to attempt courses in aeronautics or navigation in the high school given either by teachers not fully qualified to teach them or taken by pupils who have not the necessary foundations for their successful study.
6. Modify your subject matter and your rate of teaching to attain complete understanding and you will best fit boys and girls for both peace-time and war-time uses of mathematics.
7. For the present juniors and seniors in high school who have not had the regular mathematics, give an abbreviated course immediately, similar to that suggested by the Army Aviation Corps.

High School Mathematics in Artillery Fire

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It is becoming increasingly evident that sound knowledge of the mathematics usually taught in the elementary and secondary schools is an essential part of the equipment of large numbers of men and women who wish to take their proper places in our present day world. This is a fact which could have been established before the questions of Defense and War efforts arose. But the demand for men and women to do industrial work in connection with the war, and for properly prepared personnel by the armed Services, has served to call the importance of mathematics strikingly to our attention.

It is, of course, obvious that large numbers of men must have a highly technical education including mathematics far beyond the usual four-year high-school course. What is sometimes forgotten is that there is an urgent need for great numbers of men who may not require this more advanced training, but who are of no use without a knowledge of the fundamental mathematics taught in the secondary schools. It is not without reason that the complaint comes from naval officers that of ". . . 4200 entering freshmen who wished to enter the Naval Reserve Officers' Training Corps, only 10% had already taken elementary trigonometry in the high schools from which they had graduated. Only 23% of the 4200 had taken more than one and a half years of mathematics in high school."[1]

This situation must be met in some way, and it may be necessary to provide accelerated and, consequently, abbreviated courses of mathematical study to furnish some sort of training in a hurry. But it seems evident that the best prepared candidates for more advanced study, for industrial positions, or for officers' training in the armed Services will be those who have followed a sound program of substantial mathematics in the elementary and secondary schools. The purpose of such a program has been, and must continue to be, to give a thorough knowledge of *basic mathematics*, not simply an acquaintance with a scattering of applications. It is the basic *understanding* of the subject which is the foundation upon which a superstructure of applications can be built. It is impossible to foresee all the applications which can or must be made by any particular student, or to give him a solution for every problem which will arise. A significant statement appears in one of the manuals on *Gunnery* prepared by the Army for the instruction of Artillery Officers: "No manual can cover all possible situations. Therefore, when selecting gunnery methods, the officer must use *initiative* and *judgment*, as opposed to blind adherence to routine or drill." Similarly, in applying mathematics, one must exercise initiative and judgment. Without a thorough knowledge of basic mathematical ideas and methods, developed through long acquaintance, no one can expect to develop the judgment necessary to intelligent use of mathematics. And, without knowledge, initiative is useless and perhaps dangerous.

It is important that the teacher be in a position to supply an answer to the honest question, "Just *how* can this basic knowledge I'm acquiring be drawn upon in some practical situation?" Teachers are confronted with earnest inquiries of this sort every day. They answer in many ways, and they give many illustrations. This is proper, and it does not mean that they think nothing besides these applications should be studied. Perhaps such examples of the use of elementary geometry and

[1] Letter of Admiral C. W. Nimitz in *The Mathematics Teacher*, vol. XXXV, number 2 (February 1942), p. 88.

trigonometry in artillery fire as are given here may be suggestive to these teachers and stimulating to their students.

To begin with, it should be noted that the artillery officer deals with a subject of considerable mathematical content. It so happens that the actual determination of data and the aiming of a piece of artillery can be carried out without resorting, on the spot, to complicated mathematical investigations, proofs, and calculations. The "short-cut" methods used in the field, the routine operations, are learned by the officer in a long period of training. In this training he may learn by rote, or he may learn with understanding. The man who has learned by rote may perform satisfactorily in the field. But suppose, and the supposition may well materialize in fact, that something happens to the set of instruments upon which he is depending; the observation post is destroyed, or his communication with it is cut; or the enemy succeeds in disabling part of his equipment. The routine can no longer be carried out according to pattern. The officer must estimate various angles and distances, and he must combine them in sundry ways to get the proper aiming for his gun, or his unit at best will be ineffective and at worst may cause casualities among his own troops. If his learning has been by rote, and he has no clear understanding of the procedure—the "short-cuts" provided in the method—the result may be disaster. This kind of failure is not part of our American Tradition, but it could become so if geometry were omitted from American education. Instructors in gunnery know that the "understanding of short-cuts is still proportional to the student's geometrical perception." It is the background, the knowledge, the sense of security developed by serious study of and experience with his mathematics which contribute to the officer's ready grasp and proper handling of the kind of situation suggested.

Let us consider a few simple examples of the use of some basic mathematical ideas and methods in the computation of data and conduct of artillery fire. We concern ourselves only with the aiming of the gun: first, with pointing it in the correct horizontal direction; second, with tilting it at the proper angle with the ground—in fact, only the first of these problems will be dealt with in any detail. Clearly the first determines whether the projectile will strike to the right or left of the target, the second whether the projectile will overshoot or fall short of the target. The relative location of a battery of guns and a target is, therefore, described first in terms of direction (*deflection*), and second in terms of distance (*range*) and difference altitude (*site*). These data are known as *Basic data*.

The unit of angular measurement is the *mil.*[2] If the circumference of a circle is divided into 6400 equal arcs, the angle at the center subtended by one of these arcs has measure *one mil*. Clearly a straight angle (180°) is an angle of 3200 mils, a right angle (90°) is an angle of 1600 mils, half a right angle (45°) is an angle of 800 mils. In general, since an angle of 360 degrees equals an angle of 6400 mils, one degree is equivalent to

$$\frac{6400}{360}=\frac{160}{9}=17\tfrac{7}{9}\text{ mils.}$$

One mil is equivalent to

$$\frac{360}{6400}=\frac{9}{160}\text{ degree}=\frac{9}{160}\cdot 60=\frac{27}{8}=3\tfrac{3}{8}\text{ min.}$$

These relations can be used in verifying the computations in the examples given later in case the reader has a table giving trigonometric functions of angles measured in degrees rather than in mils.

Distances are measured in *yards*. A convenient approximation to an angle of one mil is the angle subtended by one yard at a distance of one thousand yards.

[2] See R. S. Burington, "The Mil as an Angular Unit and Its Importance to the Army" in *The American Mathematical Monthly*, vol. 48, number 3 (March 1941), pp. 188–189.

This leads to the very useful approximate relation known as the *mil relation:* $m=w/r$. Here w is the width of an observed object in *yards*, r is the distance (range) of the object from the observer in *thousands of yards*, and m is the measure of the angle subtended by the object at the eye of the observer (see figure 1). If α is an angle of m mils, then $m=w/r$. The mil relation is used as an approximation if the angle is less than 400 mils.

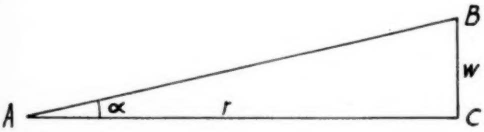

FIGURE 1

Example 1. The mil relation. Suppose in figure 1 that an object BC has width $w=24$ yds., and that it is 1800 yds. from A. Then $r=1.8$, and angle α will have measure $w/r=24/1.8=13.3$ mils.

If we assume $\angle ACB=1600$ mils (right angle), it is interesting to check the closeness of the approximation furnished by the mil relation. To do this, we note that

$$\tan\alpha=\frac{CB}{AC}=\frac{24}{1800}=0.01333.$$

A five-place table of tangents gives 13.6 mils as the measure of α. The difference is 0.3 mil. Hence the relative error in using the mil relation instead of trigonometry is $0.3/13.6=0.022$, or only about 2%.

Example 2. An object BC (figure 1) is known to be 30 yds. in width. An observer at A finds α to be 25 mils. What is the length AC (the range)? Using the mil relation we have

$$25=\frac{30}{r}\quad\text{and}\quad r=\frac{30}{25}=\frac{6}{5}=1.2.$$

Hence the range is $AC=1200$ yds.

If the reader will apply trigonometry to the finding of AC, supposing $\angle ACB$ to be a right angle, he will find that $AC=1222$ yds., an error of less than 2%.

The range of an object can be quickly estimated with fair accuracy in the field. Before going on duty, an officer determines by experiment the approximate angle in mils subtended by, say, his closed fist with arm outstretched. Similar observations give approximate angles subtended by the full span of the hand, by the index finger alone, by two fingers together, etc. In the field an object of known or estimated width w is sighted, and the angle α determined by noting that the closed fist just hides the object. The mil relation now permits a calculation of the range r. This calculation can be made almost instantly. While the approximation may be rough, it is far better than a mere guess, although experience will develop good guessing. A shot can now be fired, its point of striking observed, and a correction for the range made—if necessary!

We proceed to the matter of pointing the gun in the right direction. One method of ordering a gun pointed in a certain direction is known as *reciprocal laying*. The gun (*piece*) is pointed (*laid*) so that its direction of fire will be parallel to a specified direction. The basis for the method is the well known theorem of elementary geometry that "if two parallel lines are cut by a transversal, the interior-exterior angles are equal." In figure 2 the piece is at G, and the observation instruments are at O. The desired direction is OB, and an instrument is used to measure angle α. The angle β is made equal to α.

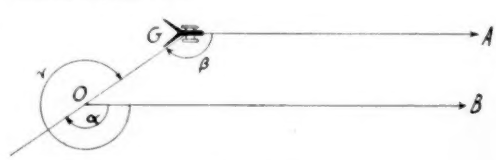

FIGURE 2

Example 3. In figure 2, the 0-3200 line (that is, the 0°–180° line) of the instrument is established in the direction OB. Suppose the angle α is found to be 3000 mils. The gunner is informed of this reading, and he uses it as a deflection and adjusts his piece accordingly. The 0-3200 line of the piece-sight (GA) is now parallel to the 0-3200

line of the instrument (OB). The operator at 0 may measure angle $\gamma = 6200$ mils. He then subtracts 3200 mils to give the reading for β. Note the importance of the basic theorem about parallel lines.

While a gun may be aimed by sighting directly at the target (*direct laying*), this is not the usual method because the target is generally not visible from the gun. In the latter case (*indirect laying*) the gun may be aimed ("the piece is laid") by sighting on a fixed point (*aiming point*) other than the target.

We take up first a simple example of indirect laying. The observation post is at O (figure 3), the gun is at G, and it is required to lay the gun on the target at T. Suppose the target is visible from O, though not from G. Let ON and GM be (parallel) lines directly north from O and G respectively. Imagine a line OS parallel to GT. If angle $\tau = \angle TOS$ can be found, then angle α can be computed at once, since it is the angle from the north line to OT plus $\angle\tau$. The gun can then be aimed by making $\angle\alpha' = \angle\alpha$.

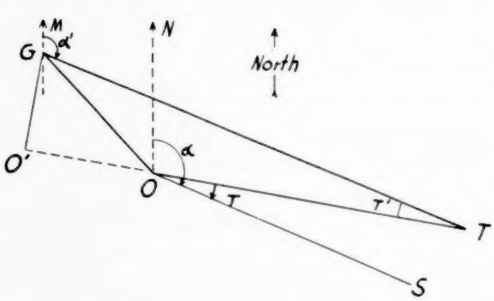

FIGURE 3

The primary task is to determine the angle τ. Since $\angle\tau = \angle\tau'$, we reduce this to finding $\angle\tau'$. This may be done as follows. First, imagine TO extended to O' so that $\angle TO'G = 1600$ mils, a right angle. In the field, the officer will probably make an estimate of the two distances $O'G$ and $O'O$. If there is sufficient time, and conditions warrant a more accurate determination of these two distances, they may be found by trigonometry. To make an accurate determination of $O'G$, we note that

$$\angle O'OG = 3200 \text{ mils} - \angle TOG.$$

Also,

$$\sin \angle O'OG = \frac{O'G}{OG},$$

and, therefore

$$O'G = OG \sin \angle O'OG.$$

Since $\angle TOG$ is measured by the instrument at O, and since OG is known (the distance from the gun to the observation post), $O'G$ can be accurately calculated. Similarly, $O'O$ can be calculated.

For
$$\frac{O'O}{OG} = \cos \angle O'OG,$$

from which we have

$$O'O = OG \cos \angle O'OG.$$

Or, it may be possible to estimate $O'G$ with fair accuracy; $O'O$ can then be found by the Pythagorean Theorem:

$$\overline{O'O}^2 = \overline{OG}^2 - \overline{O'G}^2.$$

The distance OT from observation post to target is found by measurement on a map, photograph, or chart, or measured with a range finder, estimated, or found by triangulation or other survey methods.

To sum up, we now have found by some means or other values for $O'G$, $O'O$, and OT. Since $O'T = O'O + OT$, we can use the mil relation to calculate $\angle\tau'$. For $\angle\tau' = O'G/(O'T/1000)$ mils. A still more accurate determination could be made by using the fact that $\tan \tau' = O'G/O'T$, but the mil relation is ordinarily sufficiently accurate. If $O'G$ is merely estimated, it would be foolish to attempt great accuracy in calculating $\angle\tau'$.

Since $\angle\tau = \angle\tau'$ is now known, $\angle\alpha$ is easily found by adding τ to the angle NOT read from the instrument. The result is communicated to the gunner who sets $\angle\alpha' = \angle\alpha$ on the gun sight. The gun is now pointed in the proper direction. Again, note the simple, but basic, geometry and trigonometry involved.

The range GT could be calculated by use of trigonometry, but we shall not enter

into a discussion of determination of range.

In the above discussion the customary terminology of gunnery has been avoided. This terminology will now be introduced. We first refer to the lines ON and GM. A line (or "grid line") extending north and south is called a *Y-line*. Thus ON and GM are Y-lines.

The north direction of a Y-line is called *Y-north*. The line segments directed from O to N and from G to M are each Y-north.

In order to determine a true northerly direction, that is, Y-north, a compass is used. The north direction indicated by the compass is called *compass north*. The clockwise angle between Y-north and compass north is called the *declination constant*. It is evident that with a compass and known declination constant, it is possible to determine Y-north. The clockwise angle from Y-north to a line is called the *Y-azimuth* of the line. Thus the declination constant is the Y-azimuth of compass north. In figure 3, $\angle\alpha$ is the Y-azimuth of OS, $\angle\alpha'$ is the Y-azimuth of GT, $\angle NOT$ is the Y-azimuth of OT. The horizontal angle, with vertex at the target, between the gun and the observation post is called the *target offset*. In figure 3 this is angle $GTO = \angle\tau'$.

If the reader will review carefully the discussion given about figure 3, he will see that we sought to compute the Y-azimuth of GT (that is, $\angle\alpha'$). To do this we found $\angle\alpha$ the Y-azimuth of OS, a direction parallel to GT, as follows: we measured the Y-azimuth of OT ($\angle NOT$) on the instrument at O; we computed the target offset $\tau'=\tau$; we added the offset (with proper sign) to the Y-azimuth of OT. The result was $\angle\alpha$ which equals $\angle\alpha'$, the required Y-azimuth of GT. The computation of the target offset τ' was made by using the mil-relation and an estimated or computed length $O'G$ as shown in the figure and explained above.

Example 4. In figure 3, suppose the distance OG from observation post to gun to be 1000 yds., the Y-azimuth of OT to be 1300 mils $= \angle NOT$, the angle TOG to be 2000 mils, and the distance OT to be 2200 yds. What is $\angle\alpha'$, the Y-azimuth of GT?

We follow the plan of solution outlined above. It is first necessary to compute the target offset $\angle\tau'$. This requires $O'G$. Since $\angle TOG = 2000$ mils, $\angle O'OG = 3200-2000 = 1200$ mils.

Hence
$$\begin{aligned} O'G &= OG \sin \angle O'OG \\ &= 1000\cdot\sin 1200 \text{ mils} \\ &= 1000\ (.9239) \\ &= 924 \text{ yds.} \end{aligned}$$
Also
$$\begin{aligned} O'O &= OG \cos \angle O'OG \\ &= 1000\cdot\cos 1200 \text{ mils} \\ &= 1000\ (.3827) \\ &= 383 \text{ yds.} \end{aligned}$$
Therefore
$$\begin{aligned} O'T &= O'O + OT \\ &= 383 + 2200 \\ &= 2583 \text{ yds.} \end{aligned}$$

To use the mil relation, we want $O'T/1000 = 2.58$. From this we have $\tau = (O'G/2.58) = (924/2.58) = 358$ mils. Hence the target offset is $\angle\tau' = 358$ mils. Adding this to $\angle NOT$, the Y-azimuth of OT, we find
$$\begin{aligned} \angle\alpha &= \angle NOT + \angle\tau \\ &= 1300+358 \\ &= 1658 \text{ mils} \\ &= \angle\alpha', \text{ the } Y\text{-azimuth of } GT. \end{aligned}$$

We shall consider one more example of indirect laying involving the use of an *aiming point*. The fundamental idea is this: an observation post O is set up from which a target T and some other point P (aiming point) can be seen. The gun G is located where the aiming point can be seen, but the target cannot be seen. The observer at O tells the gunner at G the angle between his line of sight GP on the aiming point and the required direction GT to the target. The gunner lays his piece on the target by use of this clockwise angle TGP (see figure 4) known as the *firing angle*.

Suppose the situation is as shown in figure 4. The observer at O sights on the target T and the aiming point P and measures the clockwise angle $\mu = \angle TOP$. The target offset and the *aiming point offset*

$\angle\theta$ are computed as will be indicated presently. The firing angle is then $\phi'=\phi=\mu+\tau+\theta$. The target offset and aiming point offset can be computed by the mil relation as explained in the discussion of figure 3. The angles τ and θ are *added* to μ in the case shown in figure 4. Other configurations will require $\phi=\mu+\tau-\theta$,

$$\phi=\mu-\tau+\theta, \quad \phi=\mu-\tau-\theta.$$

Or, the relation $\phi=\mu+\tau+\theta$ can be used if τ and θ are regarded as negative under certain circumstances. The reader can easily investigate these cases for himself.

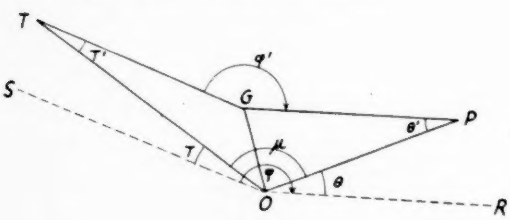

FIGURE 4

Example 5. Suppose that in figure 4 the distance from gun G to observation post O is $GO=1000$ yds., and that other data are as follows: $OP=2000$ yds., $OT=2400$ yds., $\angle TOG=900$ mils, $\angle POG=1700$ mils, and

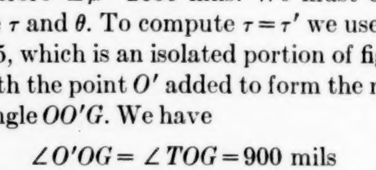

FIGURE 5

therefore $\angle\mu=2600$ mils. We must compute τ and θ. To compute $\tau=\tau'$ we use figure 5, which is an isolated portion of figure 4 with the point O' added to form the right triangle $OO'G$. We have

$$\angle O'OG=\angle TOG=900 \text{ mils}$$

Hence
$$\begin{aligned} OO' &= OG\cos\angle O'OG \\ &= 1000\cdot\cos 900 \text{ mils} \\ &= 1000\ (.6343) \\ &= 634 \text{ yds.}, \end{aligned}$$

and
$$\begin{aligned} O'G &= OG\cdot\sin\angle O'OG \\ &= 1000\cdot\sin 900 \text{ mils} \\ &= 1000\ (.7731) \\ &= 773 \text{ yds.} \end{aligned}$$

Therefore
$$O'T=OT-OO'=2400-634=1766 \text{ yds},$$
and
$$O'T/1000=1.77.$$

Using the mil relation, $\angle\tau=\angle\tau'=(O'G/1.77)=(773/1.77)=437$ mils.

Similarly, to compute $\theta=\theta'$ we use figure 6, which is an isolated portion of figure 4 with a point O'' added to make a right triangle $OO''G$. Here $\angle O''OG$ is the supplement of $\angle POG$. Hence

$$\angle O''OG=3200-1700=1500 \text{ mils}.$$

Consequently
$$\begin{aligned} OO'' &= OG\cos\angle O''OG \\ &= 1000\cdot\cos 1500 \text{ mils} \\ &= 1000\ (.0982) \\ &= 98 \text{ yds.}, \end{aligned}$$

and
$$\begin{aligned} O''G &= OG\sin\angle O''OG \\ &= 1000\cdot\sin 1500 \text{ mils} \\ &= 1000\ (.9952) \\ &= 995 \text{ yds.} \end{aligned}$$

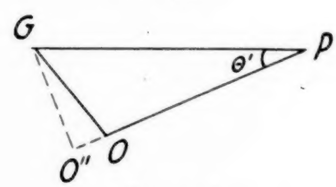

FIGURE 6

Therefore $O''P=O''O+OP=98+2000=2098$ yds. and $(O''P/1000)=2.10$. Using the mil relation,

$$\angle\theta=\angle\theta'=\frac{O''G}{2.10}=\frac{995}{2.10}=474 \text{ mils}.$$

From this we have at once that the firing angle is

$$\begin{aligned} \phi &= \mu+\tau+\theta \\ &= 2600+437+474 \\ &= 3511 \text{ mils}. \end{aligned}$$

Since τ and θ are fairly large, the mil relation introduces appreciable inaccuracies. These could be avoided by use of the tangent function instead of the mil relation. In practice, however, they would be corrected from observation of the first shot. Note that the firing angle is greater than 3200 mils (180°). The reader should draw a

diagram roughly to scale and note the configuration.

Again it is clear that simple geometry and trigonometry are all that is involved in the foregoing theory.

We have described certain simple examples of finding the direction in which a gun is to be pointed. Methods of finding range and site, and from them the elevation for the gun, will not be considered. The following diagram (figure 7) is, however, included to show certain elements of the trajectory of a projectile.

trigonometry which is involved will facilitate the officer's operations and improve his performance. The methods chosen for computing firing data depend upon the tactical situation. Where a target is stationary, the battery may be prepared and pieces laid several hours in advance. In such a case the officer will use the most refined methods available to him—trigonometric computations, and even the introduction of corrections based upon meteorological data. Here the geometry and trigonometry provide the basic results

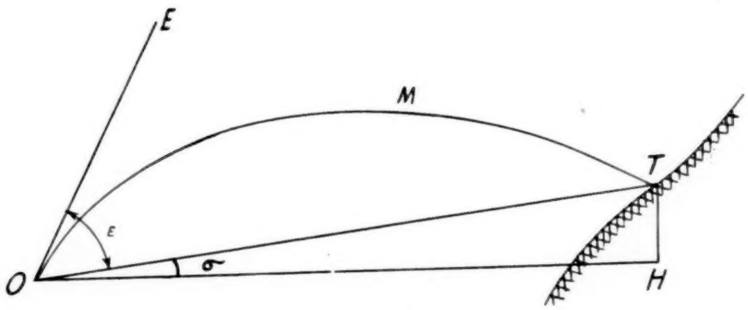

FIGURE 7

The *origin* is the center of the muzzle of the gun (piece). The point T is the *point of impact* where the projectile strikes; the line OH is horizontal, and if H is directly below T, then the distance OH is the *range*. The line OE, the axis of the bore prolonged after the piece is laid, is the *line of elevation* and it is tangent to the curve OMT, the *trajectory* of the projectile. The line OT is the *line of site* to the point T. The angle $\angle\epsilon = \angle EOT$ is the *elevation:* the angle $\angle\sigma = \angle TOH$ is the *site:* and the angle $\angle EOH = \angle\epsilon + \angle\sigma$ is the *quadrant elevation.*

The artillery officer, under circumstances of actual or simulated combat, will rely almost solely upon estimates of distances in the computation of firing data; there may well be no time for making trigonometric calculations. His ability to make accurate estimates depends, however, upon the practice and training which he has had. Part of this practice involves computing the correct distance to check the estimate. Furthermore, an understanding of the

to which any corrections are later applied.

It should now be clear to the reader that a thorough knowledge of elementary geometry and trigonometry is of the utmost value to the artillery officer. In fact, the above are illustrations of the most elementary problems which arise, and the officer will be called upon to solve problems of considerably greater difficulty. There are, naturally, many other subjects which he must know, but a geometrical sense and knowledge form part of his indispensable equipment. If he starts his specialized military training in possession of this equipment, he not only smooths the path of his own achievement, but he serves his country more promptly and more efficiently than is otherwise possible. Since December 7 we realize that in this promptness lies the hope of success in the present struggle.

EXERCISES

Four exercises are presented for the reader to try if he wishes.

1. A distant object BC of known length 120 yds. is found to subtend an angle BAC of 40 mils at an observation point A (figure 1). a) Use the mil relation to calculate the range AC. b) Calculate AC by trigonometry and find the relative error in the result of (a). *ans.* a) 3000 yds.; b) 1.8%.

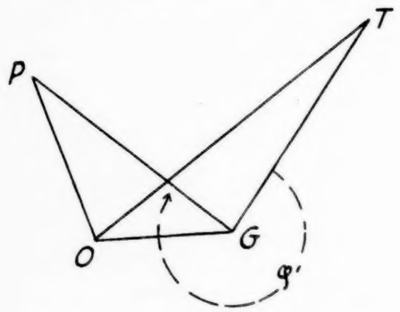

FIGURE 8

2. In figure 3, suppose the distance OG from observation post to gun to be 1200 yds., the Y-azimuth of OT to be 950 mils, the angle TOG to be 2200 mils, and the distance OT from observation post to target to be 3000 yds. What is the Y-azimuth of GT? *ans.* 1222 mils.

3. In figure 4, suppose the data to be as follows: $OG=600$ yds., $OP=1200$ yds., $OT=2000$ yds., $\angle TOG=480$ mils, $\angle POG=1800$ mils. Find the firing angle. *ans.* 2911 mils.

4. Suppose the data in the accompanying figure (figure 8) to be as follows: $GO=1350$ yds., $OP=1200$ yds., $OT=1900$ yds., $\angle TOG=720$ mils, $\angle POG=1800$ mils. Find the firing angle a) using the mil relation to compute the target and aiming point offsets; b) using a table of tangents to compute these offsets. *ans.* a) 5417 mils; b) 5373 mils.

The Coming Revolution—in Mathematics

By C. O. Oakley
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Few people, I am sure, would have had any question as to the significance of that arresting dash in the title of this paper even some months ago when the title was sent in. It clearly was not intended to be a subtle way of predicting our own entry into the war. But now that the whole world is in the struggle, it is abundantly evident that that little dash calls attention to the fact that *one* revolution is in full swing.

And one revolution at a time is enough. But although I am willing to admit that, so far as I know, prophecy does not run in my family, yet I do want to talk about another revolution that I believe is coming, a revolution in mathematics, yes even in and among mathematicians. This one is definitely not here but it is nearer than just around the corner. In fact the preliminary stages have already been completed; they started near the turn of the century when, gradually, America took up the task of developing her own graduate mathematical work. Before that time few places in this country offered substantial advanced training in our field and most graduate students went abroad to study. But that situation has changed within the past four decades and I suppose that we can safely say that we now have in the United States the greatest collection of mathematical minds, schools, and libraries in the world. And we have the freedom to pursue our studies, a freedom that is by no means universal, a freedom that is an essential part of the thing for which we are now fighting.

Yes, the preliminary phase has passed and we are entering the major phase. For the moment I shall not try to define it but rather shall speak about some of its aspects and shall indicate how the preliminary stage set the problem up. However, so as to make clear just what it is that I am talking about, I might say at the outset that I am going to discuss the coming industrial revolution in the production of graduate mathematical students. I shall be talking principally about the situation in mathematics but much of what I have to say would apply equally well to other fields, especially to the scientific fields.

First of all let us note that during, say, the last thirty years, all schools, from the grade to the graduate, have increased tremendously both in number of plants and in student enrollment. This naturally caused an increase in the demand for teachers. This demand was met by expanding colleges and universities that graduated more students who went into the teaching profession. It is clear that the problem was, primarily, one of supply and demand and both the problem and its solution seem to be on an entirely orthodox scale especially in view of the fact that the country was in a pioneer and vigorously growing state. Only when examined in greater detail do some of the trouble producing irregularities appear. And you do not need an electron microscope to reveal that where there should stand one giant with straight and tall trunk there are two main branches to the life-giving tree of teachers of mathematics; and, further, you know without my saying so that these two branches are Education and Mathematics, each written with a capital letter. This basic bifurcation of the very source of teachers has not been a happy one but nothing can be gained by hiding the fact. It is a matter of common knowledge that on the one mathematical branch all people from the budding mathematician right on up to the full blooming flower of research feel that the branch marked Education produces little more than sap which, boiled down however much, yields only taffy. And, treed up the other branch, the educa-

tion people know that their own roots are in good earth and that the fruits of the mathematical stem, unpalatable and indigestible, are produced in the dry dead leaves of works of meaningless mathematical symbolism. It is evident that both branches are grafts on a good common stock—the democratic education of a free people. And it is equally evident that the nuts growing highest up in both of these branches are over ripe and might just as well be thrown to the monkeys. Do not misunderstand me; I am a mathematician and therefore have the bent given to me as a twig and I am glad that I am living on the mathematical branch. But let me say that a lot of the wood in these branches is softwood and yields, when run through the mills, little more than pulp for newsprint and bags for holding hot air; severe pruning must be done in both branches if they are to withstand the winds which will surely blow strong during the years immediately ahead of us, years commonly referred to as "after the emergency."

We must, however, examine the situation with more care and in more detail. I know little of the internal workings of Education and so shall confine myself to the schools of mathematics. The university graduate schools built up their own teaching staffs by inviting a number of distinguished foreign scholars to their ranks (and lately the number of such has increased) and also by keeping the very best of their own graduating students. These faculties turned to research and to graduate teaching for the most part. In order to recruit graduate students, teaching assistantships were established; there were not enough scholarships to go around and larger and larger freshman classes demanded more teachers. This combination student-teacher system, though sound in principle, often worked a double hardship on the regular undergraduate body: first the senior faculty members had but few undergraduate courses and so lost some of the personal contact with undergraduates, and second the assistants, in general inexperienced and sometimes more interested in their own mathematical problems, did not always do the very best job of teaching. Many of them taught for a living and worked for a degree. It is not surprising that the quality of undergraduate instruction dropped and that finally mathematics was no longer to be found on the list of required subjects.

But of course this is but one side of the picture and all of this did not happen over night. All the while the expanding colleges and universities absorbed the best of the Ph.D's if not all of them. And too little was done for the poorer people many of whom never finished the work for their degrees. I suppose that some of these students wandered around and eventually found their way into more elementary fields. But they could not have been very happy there for they had gone to a graduate school and had taken graduate courses in mathematics and naturally never entertained the idea of doing any teaching outside of the full-fledged college. Indeed they had had instilled into them the philosophy that anything else was beneath their dignity, that advanced mathematics at or above the college level was the only mathematics worth teaching, and that research was the goal of goals. They lacked both mathematical and moral preparation for their work. The simple fact was, and still is, that most departments of mathematics design their courses for the masters degree primarily to train material suitable for the doctorate work and the work for the doctors degree to train people in the ways of research. Is it any wonder that most of the high school and junior college teachers who wish advanced work turn now to the field of Education? High school superintendents and boards of education are rightly demanding summer work of their teachers and because the advanced mathematical courses seem in most instances far removed from the secondary field, the teachers turn and return to Education. If a teacher does do further work in Mathematics instead of Education, he often finds himself out of step with the high school program.

Now I am among the first to admit, nay even stress, that a person should know how to teach before he is finally established as a teacher (and this principle applies to all levels of work) yet I cannot admit that doing more work in teacher training beyond a necessary fundamental amount to the exclusion of further appropriate work in mathematics will ever make a better teacher whose knowledge of methods increase and whose mathematical knowledge and perspective do not. I am firmly convinced that much of the ineffective teaching being done today is due primarily to narrow mathematical horizons and there are offenders in the colleges as elsewhere. It is axiomatic that you can not teach something that you do not know.

It is not likely that the world of tomorrow will see any such great expansion in educational institutions as it witnessed in the first part of this century. College and university faculties are certainly not going to increase very rapidly (they may even decrease!) and there may come a time when new appointments will be made when and only when there are retirements or deaths. If graduate schools are to survive at all they must turn their masters and their doctors degree people more and more into the secondary fields of education; the colleges and industrial concerns will not be able to absorb them. It is as simple as that. The time is right now at hand when the graduate schools of mathematics not only should but must train most of their Ph.D's for positions below the college level. For their own good they have held themselves aloof too long from anything connected with the problems of the public school systems except to deplore their defects. This may require a great change in curriculum and will most certainly require a revolutionary change in philosophic outlook. Even today a rather good man who has completed the work for his doctorate but for whom there seems to be no college post is somewhat chagrined if he has to accept a position at, say, a junior college. That a person should feel so about it is to be deeply regretted but the responsibility for this mistaken attitude lies not wholly within himself—the attitude is a product of the existing atmosphere surrounding the universities.

To me this whole turn of events is not one to be decried but rather one to be welcomed. It seems entirely appropriate to me that one main job of the universities should be to see that more and better teachers, prepared in the graduate schools, should enter the sub-college level both public and private. Indeed the whole mathematical educational structure must hang or fall together and there is no level at which good teachers of mathematics should not be available.

Let me summarize these somewhat rambling remarks into a coherent set of statements and one-man recommendations:

1. Mathematics has suffered recent attacks essentially because of the poor quality of teaching;
2. Poor teachers are poor essentially because they know too little about mathematics;
3. Advanced work in Education is inadequate for the training of teachers of mathematics;
4. Most advanced work in Mathematics train research people, not teachers;
5. A complete and adequate program of teacher training in Mathematics should be established in the existing graduate schools.
6. Local and national committees, made up of both Education and Mathematics people, should be formed now to work together on 5 which, in essence, is a common problem.

The world revolution that is now in progress will have such profound effects upon the structure of our whole educational system that the teacher training problems and difficulties which lie ahead for the graduate schools will be utterly dwarfed. But I ask you to keep firmly in mind one fact namely that the education of our people is the most important single enterprise that the United States normally undertakes.

Alternatives, Conventions and Exceptions in Mathematics

By Walter H. Carnahan
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No doubt every experienced teacher of mathematics has at times been surprised by the looks of bewilderment and doubt on the faces of the pupils when some established illusion has to be dispelled. Nor is the bewilderment always confined to looks. So often pupils come up believing that all the postulates of mathematics are necessities that all the conclusions are unavoidable, that all the conventions are like the laws of the Medes and Persians and alter not. Sometimes when an illusion is exposed pupils indicate that they think some one is playing a low trick on them, either the one who allowed them to form incorrect conclusions or the one who would have them modify or reject these. Of course no one is playing a trick. The first steps in learning fundamentals of mathematics cannot be accompanied by remarks concerning exceptions and restrictions, and the later steps in learning must take account of these conventions. The simple fact is that there comes a time when attention must be given to the restrictions and it will have to be done as considerately as possible so that the pupils do not lose all faith in mathematics. It would seem that teachers should begin calling attention to the restrictions of mathematics earlier than is now done and follow this up consistently. It is the purpose of this article to call attention to a few of the conventions and restrictions with which the pupil should be acquainted before the end of his high school course.

Pupils entering high school usually have had some introduction to the ideas of algebra, enough to understand in a general way what literal numbers are, and to know for example, that xy means x times y. It is interesting to write on the board several expressions like ab, rs, pq and tv and have pupils give their meanings, then slip in the number 23 and ask what that means. The force of inertia may cause some boy to say "That means 2 times 3." More often, however, the response is the obviously correct one, "That means twenty-three." Now try to get a statement that will explain why a 2 and a 3 written with no sign between them mean twenty-three and not 2 times 3. 23 means 2 times 10 plus 3, not 2 times 3. xy means x times y, not x times ten plus y. In each case, convention alone gives each expression its meaning, not necessity, not a law of nature, not logic. And it seems worth while to point this out to the pupil as a means of helping him understand the part that convention plays in mathematics as well as a means of helping him understand the nature of our number system. If the course in algebra later includes some digit problems, this knowledge will prove very helpful.

Try dictating this problem to your mathematics class, grade school, high school, or college; I believe the result will be about the same wherever tried: $1+2\times 3$. Of course, the result is 7, not 9. A convention of long standing takes care of these operations, but few people seem to know about it. It is pure convention and might well have been other than it is, but there is no gain in quarrelling with it or trying to change it. Time seems to have fixed it permanently. Of course, if you give the expression $x+yz$ and ask that it be evaluated for $x=1$, $y=2$, $z=3$, few mistakes will be made, which indicates that some unconscious understanding of the convention does exist.

In connection with this convention, it is interesting to recall that some few years ago two texts in algebra came out giving rules for operation in conflict with the time sanctioned convention. There could

be no logical objection to these revised rules, but convention proved too strong, and I believe later books by the same authors returned to the accepted convention.

About the fourth or fifth week in his course in Plane Geometry, the pupil encounters the Parallel Postulate. After agreement as to the definition of parallel lines, the procedure may be something like this: The teacher draws a straight line on the board and places a point above it. "Now, John," he says, "how many lines do you think can be drawn through the point parallel to the line?" John says, "One." The teacher says, "What do you think, class?" "One," they agree. "Very well, then, we shall accept it as a postulate that one line and only one can be drawn through a point parallel to a given line," says the teacher. At least, that is about the way it is done in the writer's classes. We dispose of the Parallel Postulate in three minutes. And when one considers the centuries through which the greatest logicians labored over this postulate, I think it must be admitted that three minutes is a very good record. When are pupils to be invited to consider the possibility that this postulate may not in fact accord with the science of our universe? When are they to be acquainted with the fact that non-Euclidean geometries exist that are logically quite as sound as Euclidean geometry? When can it be better done than at the time they first consider the Parallel Postulate? It is a postulate and no more; it is not the only possible one; and it is not necessarily the best one. I believe the high school sophomore who realizes this is better educated than the one who does not. And I believe mathematics presents broader and more inviting fields to him.

Probably 999 students in 1000 would say that every algebraic expression that has an x and a sign of equality in it has a root. He may admit that he cannot always find it, but he will have full confidence that it can be done. He may even have read or heard of the Fundamental Theorem of Algebra and try to quote it in justification of his belief. Of course, the reason for his unlimited confidence is that the problems in his textbooks have all been carefully selected to avoid any that have no root. Is it not a desirable part of his education to meet at least one that has no root? Try this on your class in second course high school algebra: $\sqrt{x+1}+\sqrt{31-x}=0$.

The pupil will transpose: $\sqrt{x+1}=-\sqrt{31-x}$.

He will then square: $x+1=31-x$.

Finally he will get: $x=15$.

And there he will rest the case. Now ask him to check. He will then be prepared to understand that not every expression set up to look like an equation is one. Such an expression as that above can best be described as a question which asks, "Is there a number x which satisfies the given condition?" rather than as a command which says, "Find a number x which satisfies the given condition."

Here are a few other expressions of the same kind.

$\sqrt{13-x}+1=1-\sqrt{x+3}$.

$7-\sqrt{2x+8}=\sqrt{24-2x}+7$.

$\sqrt{5x+3}+\sqrt{23-5x}=0$.

Or, with a class of freshmen, try this: $2x-4=x\left(2-\dfrac{3}{x}\right)$.

A constant source of danger is the tendency to state rules and laws without mentioning limitations and exceptions. This gets the advantage of simplicity but may lead to later confusion. For example, we state that $\sqrt{a}\sqrt{b}=\sqrt{ab}$ and often do not qualify this, as we should do, by pointing out that this holds only if at least one of the radicands is positive. Given the fundamental law concerning the product of two radicals with no limitation imposed, and confronted with the request to simplify the product $\sqrt{-4}\sqrt{-9}$ the pupil would naturally write $\sqrt{-4}\sqrt{-9}=\sqrt{-4\times-9}=\sqrt{+36}=6$. The teacher then writes $\sqrt{-4}\sqrt{-9}=\sqrt{-1\times4}\sqrt{-1\times9}=\sqrt{-1}\sqrt{4}\sqrt{-1}\sqrt{9}=i\sqrt{4}\,i\sqrt{9}=i^2\sqrt{4\times9}=-6$. The pupil wants to know what logic makes the latter result correct and the former re-

sult incorrect. The answer is, it is not logic but convention which rules here as in so many other situations in mathematics. Naturally, convention is not wholly capricious and arbitrary; there are certain conditions which make one form preferable to another, but the considerations are not wholly logical ones. (May we say in passing that the convention illustrated above has nothing to do with the sign before the radicals, as is sometimes said. $\sqrt{-4}$ means $+\sqrt{-4}$, never $-\sqrt{-4}$, just as x means $+x$ always.)

Here are other illustrations of the necessity for conscious observance of restrictions in the applications of the simplest processes of algebra. Every pupil knows that $1/(1-x)$ means "divide 1 by the expression $1-x$." He proceeds to divide and gets $1/1-x=1+x+x^2+x^3+x^4+x^5+\cdots$. Further he has been taught that x represents any number. Let $x=-1$. We then have $1/(1-x)=\frac{1}{2}=1-1+1-1+1-1+$. Now, the sum of $1-1+1-1+1-1+\cdots$ is 0 or 1, depending upon where we stop; it is never $\frac{1}{2}$. The great mathematician Leibnitz tried to justify $\frac{1}{2}$ as the correct result, and the greater Euler accepted it, but it is still incorrect. The high school pupil will see that some restriction on the indicated division must be made. He will readily agree that x cannot equal $+1$ because the divisor would then be 0; and, although he understands nothing about divergent series, he will no doubt concede that x cannot equal -1. He may even go further and find other values of x which must be excluded.

Or again: The pupil learns the application of Newton's Binomial Theorem. $(a+b)^n=a^n+na^{n-1}b+\frac{1}{2}n(n-1)a^{n-2}b^2+\cdots$. Let $a=1$, $b=-2$, $n=-1$. We have $(1-2)^{-1}=1+2+4+8+\cdots$. But $(1-2)^{-1}=(-1)^{-1}=1/-1=-1$. Hence $-1=1+2+4+8+\cdots$. Again it must be pointed out that the theorems of mathematics demand the restrictions which will make them acceptable.

The person who is to study mathematics more than casually must become as accustomed to its exceptions, conventions and limitations as to its logic, and should himself learn to seek out these iregularities. When he hears someone say that any positive number can be used as the base of a system of logarithms, he should at once supply the exceptions; only real numbers can be used, and of these 0 and 1 must be excluded. When some one says "Any number raised to the power zero equals 1," he should at once think of the exception; zero to the power zero does not equal 1. When some one says, "No negative number has a logarithm," he should correct the statement to be "No negative number has a real logarithm." For $log_e-1=i\pi$; $log\ -2=log_e2+i\pi$, etc. And when he hears some one say, "If equal quantities are divided by equals, the quotients are equal," he should at once supply the qualification, "But division by zero is always prohibited."

When he first becomes conscious of the large part played by convention and choice of alternatives in mathematics, the pupil may well ask, "Is mathematics, then, more logical than politics or theology, and, if not, wherein is its advantage over other subjects?" The answer to the first question is a matter of opinion, and the writer's opinion is that the science of mathematics is no more logical than the *science* of religion or the *science* of politics. The logic may be different, and the mathematician may handle his logic more carefully than the political economist or the theologian, but basically logic is logic whether it be employed in one subject or another. Let it be repeated that this is only the opinion of the writer. As to the second question, its answer is easier and less controversial. The great advantage of mathematics over other subjects is that it is more consistent. A convention or a postulate once laid down and accepted is everywhere and always binding unless deliberately and with good reason changed or discarded. It is this consistency as well as its logic which makes mathematics the beautiful, powerful and reasonably noncontroversial subject it is.

A Proper Balance Between Social Arithmetic and a Science of Arithmetic

By E. A. Bond

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I am downright glad to be given this chance to think with you for a few minutes about a subject that has long been nearest my heart. For more than thirty years much of my time and most of my work has been devoted to the problem of improving instruction in arithmetic. Yet at this present moment, I see more real problems ahead than at any previous time during my study. During these years innovations have come and have gone. Some of these changes have made real contributions towards improving the teaching of arithmetic. Others have had to be lived down as have other fads of education.

Let me review very briefly some of the changes in emphasis that have occurred:

About the turn of the century, students of the elementary school curriculum found that many of the processes of arithmetic had been extended far beyond the uses which they were serving at that time. They found also that many other techniques which formerly had served useful purposes were no longer needed because of changes in economic and social customs. Hence the recommendation was made that arithmetic learning exercises and teaching methods should be made to agree more nearly with what was being used, or at least with what profitably could be used.

At the same time the curriculum of the elementary school was in the process of being broadened in scope. Several new and important areas of instruction were added to the curriculum. As a consequence of these two conditions the time for arithmetic was greatly decreased (i.e., from about 16% of the total school time to about 10%). Hence the work in arithmetic was reduced in amount and much simplified in treatment.

In the teens of the century, there came the testing movement designed to measure the results of instruction and the relative effectiveness of methods of teaching. Gradually definite knowledge began to supercede theory as different areas were tested. But since it is far easier to devise objective tests to measure the abilities of pupils to do computational arithmetic than it is to measure other and relatively far more important values of the study of arithmetic, such tests became at first far more numerous than the others. Consequently, teachers emphasized computing to the almost complete neglect of the more important values that result from a proper study of arithmetic.

As a result, though the pupils did learn to compute, there was a decline in their abilities to apply their number skills to number needs. They could not see the number elements in complex situations which contained number elements along with other elements. "Their number knowledge," as one writer put it, "seemed to be in the air and of little use."

At the same time the abilities of the pupils to do the mechanics of arithmetic were far from satisfactory, since the learning was so devoid of meanings. Indeed in the two decades given up almost wholly to the teaching of the facts and processes of arithmetic, there had been little or no improvement in the abilities of children to do computational arithmetic while their ability to do problem solving had greatly decreased. Due to this lack of improvement in the former and real loss in the latter, and to certain other factors, curriculum makers began to feel that formal number work was undertaken too soon, and that as a consequence, learning became rote learning, which at best is shortlived, since it does not possess meanings and signifi-

cance. So number work in the first and in the second grades began to be taught incidentally. This change resulted in a wide variation in number work in these grades from little or no number all the way to a rich program of number experiences. Indeed in too many classrooms the number work became either casual or trivial in these grades. Yet investigations have shown that first grade children have definite number needs, possess the ability to do number work and enjoy using numbers in school and in outside activities when they know how to do so. However, I want to say the trouble is not with incidental number work as such. It rather lies in the fact that it too often results in unsystematic learning of number. Hence, the evidence supports the contention that incidental learning should be supplemented by directed activities to the extent needed for orderly and sufficient learning.

Finally, about ten years ago there began a crusade looking towards the teaching of arithmetic in such a way that it should have meaning for the pupils. "Understanding before practice for permanent learning" became the slogan for teaching the subject. Of course there have always been teachers of arithmetic who have consciously sought to assure that their pupils did see the meaning of what they were learning. I had at least two such in my childhood. But it is also true that many teachers neglected to see to it that the pupils learned to apply their learning to the real problems which surrounded them. Indeed research workers found that from 85% to 90% of the time devoted to arithmetic was spent upon computational arithmetic at the time.

At the present time there is a marked tendency to overemphasize the social uses of arithmetic without duly considering the mathematical setting of the learning and insight into the science of number. As a result, number relations are either neglected as unimportant or are considered on too low mathematical levels. Since we have the habit of pushing a good thing too far, this tendency will probably go too far. This probability is the reason for the title of this paper and will be the theme hereafter in it.

At the outset, I want to say, as I have said at other meetings of the Council, incidental learning has made two very valuable contributions to the teaching of arithmetic. It has emphasized children's interests as a motivating principle for effort. It has also stressed the value of concrete illustrative material for giving meaning to learning. Both of these contributions are valuable and needed. Both should be kept and enhanced. The meaning of what is being learned both increases the opportunity for recall and at the same time strengthens the significance of what is being learned. Both of these results help children beget attitudes favorable to learning. Furthermore, favorable attitudes are central to economic learning. But learning must be developmental.

There are then two very different but mutually helpful possible objectives for teachers of arithmetic. The one has for its aim the development by the pupil of a unified science of arithmetic. This has been called the mathematical aspect of the teaching of this subject. The other looks towards the uses of arithmetic in satisfying the quantitative needs of the social and economic life of our people. This is called the social aspect of the teaching of arithmetic. They are both needed. Indeed they are both necessary adjuncts of a power to use number to advantage. On the one hand one cannot use knowledge that he does not possess. On the other one cannot use the skills that he possesses unless he understands their significance in attacking situations that contain quantitative aspects.

At the present time it seems that there is not a proper balance between the two objectives. As has been said, they are both needed. The social uses now are receiving the major part of the attention of teachers of arithmetic. The reason for this condition is obvious. Until but a few years ago

this phase of the teaching of arithmetic received far too little thought and attention. Now we are going toward the other extreme. It is true that arithmetic was introduced into the curriculum because of its universal need in our business and social undertakings. Then due to its inherent interest, it outran the purposes for which it was introduced. Then it came to be taught largely for its own sake.

Now the reaction is in the other direction. A large number of careful investigators hold that a unified science of number cannot be built from the social phases alone. But on the contrary the social uses of number provide the best illustrative material through which the science can be built in a meaningful way. So it seems that the best procedure is a movement back and forth between the concrete problems of society and the general principles underlying the quantitative illumination of these problems. This is especially true if we want our pupils to develop the power to attack other socially important problems successfully.

Undoubtedly, the most desirable end result of the teaching of arithmetic is the development of a science of number clarified by its uses to the extent that it is applicable to any situation that contains number elements. The science should contain only those needs that well informed citizens of our time have. So conceived arithmetic is a mode of thought accompanied by knowledges which when acquired will enable its possessor to see the number elements in a complex social situation, to analyze these number elements and to give number its just weight in responding to the whole situation. The lack of the number-mindedness implied above is obvious on every hand. Millions of our workers have budgeted their incomes beyond their limits with obligations incident to installment buying and to personal loans, the financing charges of both of which are out of proportion to the services rendered. Again there are large numbers of endowment life insurance policies sold to those whose primary needs are solely for protection. In both of these illustrations and in many other situations general quantitative knowledge coupled with the habit of making quantitative judgments would be useful.

Annual Meeting for 1943 Cancelled

The Annual Meeting of the National Council of Teachers of Mathematics, which was to have been held in St. Louis in February, has been called off, because of transportation difficulties and other important reasons connected with the war effort. Because of the importance of mathematics teachers getting together in these eventful times to discuss their problems and particularly to discuss the role of mathematics in relation to the total war situation, President Rolland Smith is encouraging members of the Council to plan to hold regional conferences in large centers such as New York, Chicago, and St. Louis.

An all day New York meeting is to be held on December 31, 1942, at the Horace Mann School Auditorium of Teachers College, Columbia University, at Broadway and 120th Street. The program for this meeting will appear in the December issue of The Mathematics Teacher. This meeting will follow a 3-day session of the A. A. A. S. on December 28, 29 and 30 in New York City. The St. Louis meeting will probably be held in February and the Chicago meeting in March. Further notices about these and other regional meetings will appear in later issues of The Mathematics Teacher.

A 75¢ luncheon will be served for those who wish to attend on December 31, at the Teachers College Cafeteria. Reservations for this luncheon should be sent to W. D. Reeve, Teachers College, 525 W. 120 St., before December 25.

Arithmetic as Concept Building*

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Before we can think together clearly and profoundly, we shall have to agree what we mean when we use the term arithmetic. Arithmetic has been commonly conceived as synonymous with number. This point of view has led to arithmetic's being developed as a tool and consisting of processes of manipulation of symbols rather than its being considered a process of intelligent derivation and application of basic concepts with regard for quantity. It has led to the postponement of so-called arithmetic in the first two primary grades and is largely responsible for the dislike of arithmetic as a subject. The concept that arithmetic is number manipulation has led to the development of accurate and speedy computers who are intellectually imprisoned and incapable of making application of their mass of symbol-habituations. Is this what we mean by the term arithmetic or do we mean something much more fundamental and functional? If our major concern is behavior, are we not vitally concerned with the interpretation of stimulus-patterns that lead to a maximum degree of insight into quantitative relationships and to the formulation of functional systems of concepts? Such behavior results in significant understandings for intelligent adaptation to one's environment and for the creation of more effective quantitative relationships.

I should like, for a few minutes, to develop a definition that has been given widespread consideration through its development by Dr. Peter L. Spencer of Claremont Colleges. The concept is that "arithmetic consists of the process of making discriminative reactions with regard for quantity and relationships among quantities."[1] Stimuli for discriminative reactions exist in all awareness because everything within the environment exists as quantity. We are continuously reacting with regard for quantity, and are, therefore, continuously growing in our sensing of relationships. From behavior in sensing relationships ideas are created, and ideas expand into concepts. The refinement of ideas into systems of concepts is a process that is continuous with each individual from the beginning of behavior until the end of behavior. The degree of insight which one has into his system of concepts determines his behavior with regard for his quantitative environment.

Arithmetic from the broad viewpoint permeates all subject-matter and has potentialities for significant control and betterment of society. As a process it involves reading of quantitative phenomena in all fields. The systems of order and techniques for expression constitute an outgrowth of understanding of the basic concepts of each field of regard. There is a significant difference between the reading of quantitative phenomena and the memorization of symbolic facts and skills. For example, time-telling, memorization of the days of the week and the months of the year results in mechanical fluency with the symbols under consideration. However, it does not lead to insightful behavior that develops through an understanding of the concepts of time that are of major concern for social communication and for intelligent interpretation of time. Intelligent understanding of time leads to an appreciation for the measurement of time and to interpretations which facilitate communication and implement desirable temporal relationships.

* Paper read at the annual meeting of The National Council of Teachers of Mathematics in San Francisco, February 1942.

[1] . . . A concept for arithmetic that has been given widespread consideration through its development by Dr. Peter L. Spencer.

Another example of difference between acquired symbolism and intelligent application of basic concepts with regard for quantity is that of sensing economic values. Economic relationships are basic problems for each individual and for society as a whole. The power of the consumer through his support of undesirable industries by unwise expenditures and the development of desirable industries through wise expenditures is a matter of our concern. Sensing of economic relationships in an efficient way raises the level of living of the individual and concomitantly of society. Desirable economic status cannot be acquired through having individuals memorize so-called three types of percentage, budget plans, what to turn upside down in the division of common fractions, and other techniques for the expression of measurement. A bulletin, *Credit Problems of Families*, published by the United States Department of the Interior, gives some interesting figures concerning installment buying. A survey made in 1937 by the United States Department of Commerce revealed that "about 91% of household appliances, 92% of lumber and building materials, 91% of furniture, and 66% of new automobiles sold in the United States were bought on open account or on the installment basis. . . . Another study made by the same Department in 1937 indicated that about one-half of owner-occupied urban homes were mortgaged."[2] Installment buying has increased considerably since the first World War. The use of credit has been wise in many cases and tragic in many others. Inadequate sensing of values has caused many families to lower their level of living. A real challenge to educators lies in stimulating the intelligent sensing of values and the insightful application of quantitative relationships so that more effective economic relationships will result.

[2] United States Department of the Interior, *Credit Problems of Families*. (Washington, D.C.: United States Government Printing Office. 1940.)

Similarly, there is a challenge with regard for spatial orientation. For example, map-reading as an aspect of spatial orientation, is often developed in a way such that symbolism rather than intelligent control is the product of teaching. Frequently children are taught to identify north, south, east, and west, and yet have little insight into map-reading and reacting to spatial patterns. There is an exceptional opportunity to develop basic quantitative concepts with this subject.

Only as society understands its systems for measurement and techniques for expression of measurement is it ready to make changes that will lead to more effective communication. The need for change becomes evident when one examines much of our accepted symbolism. For example, much confusion arises with many of our number names. Children have considerable difficulty in sensing the positional value and meaning of the number names in the "teens" decade. The difficulty arises largely because such number names do not reveal clearly the principles of the number system. How much simpler and more functional would be such terms as onety, onety-one, onety-two, et cetera. One names the digit to the left while "ty" indicates its positional value of ten. The unit's part of the symbol is represented by the basic values for the terms in the primary series. Changes of these and of certain number names to more rational names that would reveal the order of our notational system would greatly facilitate communication and understanding.

We have stated that mathematical behavior is continuously developing. Teachers do not initiate arithmetic. Mathematical behavior within the child has been in a process of development a long time before he comes to school. He comes to school with a high level of conceptual development and is ready for concepts that are commonly graded for years later than the primary grades. He reacts insightfully from even the viewpoint of the adult if the adult is astute in reading the child's mean-

ing. The process of reading the child to determine his level of understanding and then pacing stimulation in a manner such that growth can be maximum is fundamental because learning is an emergent process. Whatever one conceives is an outgrowth of his inherent abilities, his experiences, and his goals. It is the task of the teacher to lead the child to recreate the order and system that the race has taken a considerable period of time to develop. In order to recreate the quantitative systems that are accepted by society, the invividual must be stimulated through quantitative experiences in a manner such that he will sense the quantitative problems which society must solve. He should avoid procedures that are inexpedient and be challenged to make discoveries that will lead to a high level of understanding of relationships which are pertinent to this quantitative environment. A major problem for all teachers is the formulation of procedures to stimulate the development of basic concepts through which symbols will emerge in a meaningful way. The mass of implementation is significant in terms of the meaning which the individual projects into it.

Many teachers—nursery school through graduate school—have erred in conceiving symbols to exist in their own right rather than sensing that symbols have only the meaning which an individual projects into them. Number and all quantitative symbolism exists only as a man-made device to control the environment. Systems of order are highly functional in reducing the environment to finer and finer terms, but in order to have intelligent control of quantity the individual must have a high level of insight into the systems for measurement. In ancient time the process of reading and writing was developed but by few people. The ordinary man could not decipher the symbols on the papyrus that denoted laws of measurement. Perhaps the situation is comparable today in schools where teachers are unable to stimulate students to develop more than number or symbol work—when their teaching does not stimulate control of the quantitative environment that has signification beyond manipulation of symbols.

Any subject that is developed in a way such that there is intellectual isolation of meaning should be questioned. One place to start such questioning is with textbooks designed for children. They are often profusely packed with exercises designed to develop efficient behavior with manipulation of number and very little reason is revealed for rules and procedures. For example, a recent textbook contains 937 examples with subtraction—enough examples to provide considerable busy work—and yet there is inadequate development of concepts with regard for bridging in subtraction and there is deficient provision for the types of subtraction examples which will occur in long division examples. One-hundred-one of the 937 examples are ones that occur more than once. For example, page 111 contains the same examples as page 110, but the arrangement of the examples on the two pages is different. Of the 937 examples, only 94 can occur with more than one divisor in division examples having one-digit and two-digit divisors. These 94 examples represent approximately 10% of the 937 examples. This busy work seems not unlike the procedure of secret societies formed by the pupils of Pythagoras who surrounded mathematics with mystery and secrecy! Remember Hippasus was reported to have been drowned in his bath for divulging mathematical truths outside his society! I think that we agree that teachers rather than textbooks should be the main source for stimulation and direction of concepts. However, unless teachers are educated to teach without textbooks, the textbook retains a pertinent position. The creation of textbooks that contain materials that are conducive to significant understanding is a profound challenge to writers of arithmetic textbooks for children.

How can we stimulate children so that they have significant insight into arith-

metic? Insight does not emerge with mere duration of time. Sensing color relationships, fine proportion of window area, how long fifty years is, how big a whale is, how far a mile is, the size of our defense expenditure, are all examples of concepts that develop through purposeful stimulation and sincere thought. There has been a tendency on the part of teachers and writers to hold somewhat tenaciously to the idea that children cannot understand many topics listed in arithmetic books, and that the solution is to postpone or eliminate the things that offer difficulty. This idea has been expounded with very little illation and questioning of how concepts develop and how thinking with regard for them can be motivated. I have been working in a very direct way with children in the primary grades, and I find readiness for sincere thought and ability within children to comprehend in a dynamic way concepts that are often inadequately developed in adults. I have failed to find children in the seven, eight, and nine-year-old group who are unable to understand and interpret decimal expressions and basic concepts with regard for quantity. By the term decimals I refer to whole numbers. Decimal fractions occur with the extension of the decimal system of notation to the right of the units' position.

To be more specific, I should like to discuss teaching the development of our decimal system of notation. Relatively few teachers stimulate children to have a high level of understanding of our decimal system. The symbolism that the work decimal comes from the Latin word "decim" meaning ten may be said, but it is an inanity unless children are stimulated to understand decimals so that they have insightful control with the placement of decimal points in work with decimal fractions—including work with division of decimal fractions. By insightful control I am not referring to the memorization of rules for counting decimal places. Insight into the decimal system of notation involves an understanding of the inherent or constant and the positional value of number. The inherent or constant value of each primary digit refers to its rank in the series of primary digits. Zero, 1, 2, 3, 4, 5, 6, 7, 8, and 9 each has an inherent value. Each digit derives its meaning from its position in the series. Six, for example, is always one more than five, and one less than seven. It is three twos, two threes, six ones, et cetera. Before a child can rationalize his procedures with numbers, he must understand their constant or inherent value. An apparent lack of insight with number combinations may be partly attributable to deficient expansion of this concept. As numbers larger than nine are used, positional value should become apparent. The discovery of the positional value of numbers should develop in the child a deep appreciation for the decimal notational system. Various procedures may be used to facilitate maturation of the child's concepts with inherent and positional value. The discovery that in the numerical expression 444, the first four at the left has a value of 400, that the middle four has a value of 4 tens or 40 and that the first four at the right has a value of 4 only can be refined and expanded. The child can be stimulated to discover that the middle four is ten times as valuable as the four at the right of it, and that the four at the extreme left is ten times as valuable as the middle four and one hundred times as valuable as the last four at the right. The discovery that placing a figure one place to the left of its former position makes that figure ten times as valuable, and that placing a figure one column to the right decreases its value ten times is a differentiation that is significant.

The problem of writing a number ten times as small as the digit at the extreme right of the given numerical expression presents the need for a decimal point to mark the point where the whole numbers stop and the decimal fractions begin. The need for the extension of the decimal system to the right of the units' position can very well and does arise after the child un-

derstands positional value with whole numbers. Children in the primary grades can readily understand this extension of the decimal system. Their knowledge of odometers of automobiles is an example of their readiness for this concept.

The logical expansion of concepts inherent in our systems for measurement should be the basis for the selection of methods and procedures of all aspects of manipulation. Methods that are selected from the standpoint that research indicates one method to be superior to another may be questioned. Often the results of research indicate the superiority of a specific procedure because that procedure was found to have given, over a short period of time, superior speed and accuracy with manipulative skills. A more pertinent criterion is which method logically expands the concepts which are inherent in the whole pattern of quantitative relationships. This is an especially pertinent issue for the State of California at the present time. A set of new textbooks were released not long ago, and they are receiving considerable questioning from this viewpoint. Evidence of this questioning may be seen in an editorial in the *California Journal of Elementary Education*, May, 1941.[3] There is considerable concern about the adoption of the equal additions method for subtraction. This questioning, in my judgement, is justified and should lead to further analysis of methods.

I should like, for the remainder of my time, to have us think into the logical expansion of the concepts with the division process. Does it exist as a process in its own right? A functional concept for division has been in the process of development a long time before the child comes to school. It is the symbolism of our technique for expression of the concept that retards the child. The teacher who astutely reads the maturation of his children and who has the ability to motivate children to discover that long division is a special case of subtraction, releases possibilities for insightful behavior that should lead toward appreciation for and significant control of performance with long division. Division as a special case of subtraction consists in determining the number of times the divisor, which is the subtrahend, can be taken from the dividend which is the minuend. The quotient shows the number of times the subtraction has been made. The remainder indicates the amount which is left unsubtracted.

Even at the college level students frequently have acquired the mechanics of the division process without sensing the above relationship, and similarly relationships with the divisor, dividend, and quotient. Rules concerning divisor, dividend and quotient relationships are of questionable value unless they have been derived by the learner.

In conclusion, a problem of primary importance for teachers is that of stimulating children to understand basic concepts so that they will have intelligent control of their quantitative environment. A secondary concern is the stimulation of children to acquire the techniques for implementation and expression of the basic concepts. Thus, children should not become automatons who are mechanically fluent with tables and number facts, but they should have insightful control of their behavior. "Arithmetic as a process of making discriminative reactions with regard for quantity and relationships among quantities" has potentialities for significant interpretation and control of the environment.

[3] Editorial, "Equal Additions Method of Subtraction," *California Journal of Elementary Education.* Vol. 9: 193–196, May, 1941.

National Council Members !

Be sure to attend at least one of the regional meetings of the Council this year.

Socializing Secondary School Mathematics

By Ralph D. Shamhart
Junior High School, Prescott, Arizona

We are living in a world which is incurably mathematical. Everyone knows it. The simplest article of manufacture is subject to the rules of mathematics in production and sale. To list the uses of the subject would be impossible and futile. Yet mathematics has come under criticism as a subject to be taught in our schools.

Perhaps one cause of criticism is that mathematics has the misfortune of being one of the oldest school subjects. Euclid wrote a very decent geometry treatise about 300 B.C., even judging by our "progressive" standards! By modern educational standards all that is old is worthless so—throw out mathematics!

Another cause of criticism springs from the fact that our methods of teaching mathematics are obsolete. "Small fry" in the teaching profession have consistently transferred criticism of their poor methods to criticism of mathematics. When, as in a faculty meeting, their subject comes under fire, they sit in apologetic silence offering no defense or explanation.

It is true that in this world ruled by the principles of mathematics our methods of teaching it are scarcely past the horse-and-buggy stage. Obviously it is not the subject which is at fault—it is we who teach it. Let us have a change, a change in method of teaching.

Criticism, as such has no value unless it is constructive. With this in mind let us examine mathematics to ascertain the following:

1. What is to be derived from mathematics? What is its worth?
2. How can we best present mathematics to insure the assimilation of its desirable materials?

To answer the first question let us state four objectives. Mathematics should:

1. Enable pupils to deal successfully with such quantitative situations as they are likely to meet in their everyday experiences; particularly, in their school studies.
2. Prepare students, as far as it seems feasible, to meet quantitative situations in adult life.
3. Give them an understanding of the meanings of the social uses and applications of mathematics.
4. Develop a high degree of familiarity with, and accuracy in, the fundamental processes of arithmetic.

These are rather generalized aims of teaching arithmetic and mathematics. There is an increasing demand that secondary schools narrow and confine their courses to conform to them.

From the tremendous field of mathematics it is difficult to choose the bare fundamentals a child may need in everyday life. Surveys have been made to ascertain just what these are. The results show that the actual needs of the adult are very little more than intermediate grade fundamentals. Some feel this to mean that much of the unnecessary material should be discarded. Others say that it shows mathematics is not being taught correctly, for if it were it would be used more extensively. Both factions may be reasoning correctly.

Text book authors have taken care of the first complaint. Modern texts have discarded much unnecessary mathematics. The second complaint must be cared for by the teachers themselves.

Certain fundamental skills must be derived from the study of mathematics. These might be:

1. Clearness of concept. Our symbols in arithmetic must be understood clearly to be used.
2. Economy of thought and expression.

We continually abbreviate, and as we advance, our symbols are more and more inclusive.

3. Preciseness.
4. Accuracy.
5. Habit of analysis.
6. Habit of logical reasoning.

Now, how may we teach in order to attain these skills and habits?

Let us deal with number one, clearness of concept. In no other subject is it quite so important that we understand clearly as we go along. The understanding of mathematics must grow with the student as a house is built. If part of the foundation of a house is missing there can be no house until the mistake is rectified. So it is with mathematics. Nine out of ten failures in our subject can be traced to a lack of some fundamental process such as long division, decimals, or fractions. Any algebra or geometry class contains one or more students who cannot perform all of the fundamental processes. These students are the ones who fill in the lower end of our beloved grade curve. A teacher must, then, give diagnostic tests to see that each and every student has command of the four fundamental processes of decimals, fractions, and whole numbers (i.e., multiplication, division, subtraction, addition).

Our second skill to be gained is economy of thought and expression. Economy is necessary in the study of mathematics problems. However, the excessive use of symbols is sometimes regarded as one of the contributors to stagnation of mathematics. Symbols tend to overshadow everything else. Their use and the use of signed numbers and equations may deaden mathematics. Therefore, one of the prime responsibilities of the teacher is to see that each student understands and appreciates their usefulness. They are the tools and time savers of mathematics, not the stumbling blocks.

Preciseness and accuracy are basic necessities to everyone in the commercial world of today and in any phase of life. Anyone who enters a mathematics class knows that these are emphasized enough, perhaps too much.

The habits of analysis and of logical reasoning are perhaps mathematics' greatest contributions to the individual. Methods of attack on problems may be applied to any problem a person has. The habit of logical reasoning helps to form better citizens for democracy.

Every problem and process explained by the teacher should follow these steps:

1. Make sure you understand the meaning of the problem.
2. Decide what is to be found to satisfy this problem; or, what does the problem want to know.
3. What facts are given or known that might be used?
4. How can these facts be manipulated to find the answer? What formulae are known that might be used?
5. Estimate what the answer might be.
6. Try to solve the problem.
7. Does the answer sound logical?
8. Check your results.

These steps of analysis may be applied to practically any question that requires analytical reasoning. To prove this, try applying these steps to the problem from which this paper evolved—that of discussing pro and con current problems of education.

Logical reasoning is really the application of the scientific method to the process of thinking.

That the steps of analysis may be applied to any field where thought is necessary may hint at transfer of training. If this is transfer of training, then the author believes in transfer of training. However, this is transfer of method, logical method of uncovering facts unknown to the person before. It is training to make a habit of a well-organized method of attack.

One or two years of secondary school mathematics are not likely to impress upon the student the idea that the process used in solving mathematics problems is

applicable to any problem in other fields unless the teacher continuously stresses this idea. Mathematics should proceed by analysis and logical reasoning, not by mere application of formulae without knowing why it is applied.

Here are some methods to be used in teaching fundamental skills which may help to bring mathematics up to the times and to socialize it.

Each year of mathematics should begin with diagnostic testing to find the students who are weak in fundamental processes of addition, subtraction, multiplication, and division of whole numbers, fractions, and decimals. It is hopeless to try to teach secondary school mathematics to students who do not know how to divide decimals. Practically all failures can be traced to some deficiency of this sort. Discovery of cases should lead to immediate remedial teaching.

Mathematical formulas, symbols, and equations should be used only after the idea behind them or the idea from which they evolve is clear to the student. They are only shorthand methods of dealing with mathematical statements.

The instructor should justify the course at least once a week. This may be done by means of clippings which involve mathematics (practically every clipping in the newspapers and magazines has some mathematical application). Or perhaps a discussion of some profession, position, or activity and its relationship to mathematics. A very effective means of justification is a bulletin board on which pupils post clippings to illustrate the place of mathematics in all walks of life.

One criticism of mathematics teachers is that they accept 90% perfect classwork as excellent, whereas in business nothing short of 100% accuracy is accepted. The fallacy here is that in business virtually the same mathematical processes are dealt with over and over again, while in a class almost every day brings a new process to be mastered. A 90% average on classwork which means virtually a "first try" at the processes seems to me to be very good.

There are only two reasons why a child should receive a failing grade in mathematics. These reasons are excessive absence from class and absolute lack of effort. This should be made clear to the student at the first of the year. Then the fear of mathematics will be removed. Most students hate to enroll in a math class due to the fear of failures. It is the duty of the teacher to take the student where he finds him and help him advance as far as possible. Adapt the course of study to the individual student. If he doesn't know sixth grade arithmetic when he enrolls as a ninth grader, put him into a class where he can learn his sixth grade arithmetic. This can soon be detected, no matter what math class enrolls him. An infinitesimally small percentage of ordinary students absolutely cannot understand secondary school mathematics. The largest single cause of pupil failure is poor teaching methods.

Every problem and process outlined by the teacher or the student should follow a definite analytical outline as suggested in this paper.

The child should be taught to make simplified drawings to illustrate his problem. This helps to clarify the issues to the student. These drawings must not be required in the work to be handed to the instructor for diagnosis, but just as a part of his "scratch work" and for his own use.

In about every ten days of classwork there should be a holiday from any written work at all. There are on the market, many books of mathematical puzzles. These, while they are still mathematics, make the classwork more interesting. Remember, it is not an unforgivable sin to conduct one whole hour of mathematics without requiring a written paper.

Permit students to work together in a math class. Many students will ask others to explain a problem to them where they will not ask the teacher. This step requires strict supervision, however, to prevent dishonesty. Such supervision should under

no circumstances be obvious. If a child is under suspicion make it a point to take that exact time to see how the student is getting along with the assignment. If such parasitism persists, one defense of student failure is lack of effort.

While these suggested changes in mathematics teaching methods are not expected to be a cure-all for formalized methods, they will be a step in the right direction; and, above all, the suggestions are practical!

The Habitat of Geometric Forms

By Charles R. Salit
Floral Park, N. Y.

In the natural sciences like biology and botany the question of habitat constitutes a very important phase of inquiry. It is a method by which certain animal and plant forms are located and new ones are identified. In geometry, however, where one is seemingly dealing with artificial forms, the method of habitat would indeed appear to be out of place. These forms, one might contend, having no organic life of their own, may and can be disseminated almost indiscriminately and at will across the face of the earth, regardless and quite independent of environment. In short, one might conclude, the geometric form has no particular home.

Such a view, however, would seem to infer that the geometric form sprang into life full grown and full armed, so to speak, like Pallas Athena from the head of Zeus. Such a view, furthermore, would seem to ignore the all important fact that the geometric form is inextricably intertwined and bound up with human experience and can hardly be thought of apart from it. In fact we may speak of an evolution of the geometric form as one of the most outstanding phases of the spiritual and intellectual growth of man.

An inquiry into the origins of the primary geometric forms will reveal a religious rather than a utilitarian background. It will show that the primary geometric forms are nature forms that the primitive man singled out for particular distinction and religious worship. That these god-forms could be put to practical use and made to do physical work was a discovery of the principles along which our present civilization was evolved. Thus, the Peruvian Indian was a sun and moon worshipper, and, therefore, familiar with the sun and moon disks, as shown by the archaeological findings in their temples. And yet, the Incas did not have the knowledge of the wheel system. It is one thing to know that diamonds exist, but it is quite another thing to know that they can be used as jewelry and as tools for cutting hard surfaces. So too, we may know that coal exists, but it is quite another matter to know that it can be used as fuel!

In the matter of habitat of the geometric form the story of its evolution is extremely important. Though we may think of the form and shape of our civilization as purely profane and material, religious practices and ritual of long and unknown ages have woven its very warp and woof. It is not a mere chance that our church towers are in the form of a cone or a pyramid and that the obelisk is planted in the cemeteries. Nor is it a mere chance that our stately public buildings and memorials display the cylinder in the form of a column, and sometimes—the prism, in the form of a pillar! Or, why is it that in our house architecture we find the isosceles, the right, and—occasionally—the scalene triangle, but hardly if ever the equilateral triangle? Just where should one look for a square and a rhombus? Are they common figures in the fields of mechanics and engineering? art? religion? architecture?

In partial answer to these queries and as a further elucidation, it is noteworthy as well as interesting that the primary geometric forms retain their original religious meaning. By way of illustration, let us take the equilateral triangle. It is undoubtedly the oldest and, in fact, may be considered the ancestor of all triangles. And now let us consider some of its uses and trace it to its habitat.

In theology, the equilateral triangle is frequently used to illustrate diagrammatically the Holy Trinity. The double triangle of the Hebrews, known as the "Shield of David" or as the "Seal of Solo-

mon" is always displayed over the main entrance of the synagogue. Here it has a recondite allusion to the Tetragrammaton or four-lettered, incommunicable name of God. This symbol has also been used at one time or other by the various mystic orders of freemasonry. The equilateral triangle can also be seen over our Y. M. C. A.'s, ablaze with electric lights, symbolizing the physical, social, and moral aspects of man. In Gothic architecture it is the decisive element of construction. The pointed arches are all based on it. In Babylon, Greece, and Rome the equilateral triangle served at certain periods as the norm for their temples. Since religious practices are closely associated with decorative art, the equilateral triangle as a foundation for design possesses greater power of variation than any other geometric form.

Not unlike the equilateral triangle, the square too possesses certain religious and mystic properties, in particular in Hindu religion and philosophy. In the western or Christian countries, however, the square is best known as one of the most important bases in decorative art, a sure sign of its religious origin.

From this point of view, at any rate, the rhombus differs from the square only in that the former is lacking in these mystic and religious qualities, and that while the latter is chiefly a basis for design, the rhombus itself forms the decorative design. No other quadrilateral possesses these properties.

In the study of the habitat of the geometric forms it is necessary to remember that they are nature forms that man had singled out for religious worship. Civilization, however, as we understand the word, can only flourish when man consciously begins to put to practical use these forms that a religious ritual has unconsciously evolved in the course of nature worship. It is highly questionable if any people of Western Europe made any practical use of the geometric form before the infiltration of advanced ideas from the Near or even the Far East! It may not be so far-fetched to suggest Persia, Arabia, Egypt, China, and India as the countries where the primary geometric forms first flourished. This follows from the axiom that we here propose, that there can exist no civilization as we understand it without the knowledge and the practical application of the primary geometric forms. From this axiom it then follows that the primitive civilizations of equatorial Africa, and perhaps of the whole American continent could not have been highly advanced for the very lack of this practical knowledge. But here again we wish to repeat, these primitive peoples were very likely familiar with all the geometric forms in the course of their nature worship, but they did not know them all in a practical, utilitarian way.

A Guid Scotch Wish

May the best ye've ever seen
Be the warst ye'll ever see.
May the mouse ne'er lea' yer pantry
Wi' a tear-drap in her e'e.
May yer lum keep blithely reekin'
Till ye're auld enough tae de'e.
May ye aye be just as happy
As I'm wishin' ye tae be.

The Teacher's Job in Total War*

We are engaged in total war. That is why we are meeting here today. For total war reaches into every phase of a nation's life. Total war is waged not only on the battlefront, in the factory and in the homes. It is waged in every classroom throughout the nation. Every classroom is a citadel. Every teacher has his part to play. The job of the armed forces is to win this total war on the battlefront. The job of industry is to furnish the weapons and supplies needed by the armed forces to carry on total war. The job of the schools in this total war is to educate the nation's man-power for war and for the peace that follows. We can lose this total war on the battlefront as a direct result of losing it on the industrial front, on the home front or on the educational front. Education is the backbone of an army. This was never more true than it is today—now.

Our Army today is an army of specialists. Out of every 100 men inducted into the service, sixty-three are assigned to duties requiring specialized training. We aren't getting those sixty-three specialists through the induction centers. But modern mechanized warfare dictates that we must have them—men who know the fundamentals of electricity, who know automotive mechanics, who can operate radios, or dismantle carburetors. Without them, your army would be an incongruous mass, incapable of attaining any objective. How badly do we need them? How big is the deficit? Here are some figures. Herein is the crux of your Army's need for trained manpower. On January 1, 1942, out of every 1,000 men inducted, your Army needed fifteen who had some kind of training as radio operators. From February 1, 1942, through March 31, 1942, we were getting less then one man per 1,000. We were short then almost fifteen men per 1,000 inducted. Think of that! Actually out of every 300,000 men inducted, we needed 4,689 with training as radio operators. We were getting 135. We were short 4,554. Out of every 300,000 men inducted, your Army needed 4,501 with training as medical technicians. We were getting 166, a shortage of 4,335. We needed 4,372 telephone and telegraph linemen. We were getting 343, a shortage of 4,029. We needed 1,562 master mechanics. We were getting fourteen, a shortage of 1,548. In the entire field of automotive mechanics, which includes many allied subjects, out of every 300,000 men inducted, we were short 10,437. That means a shortage of 34,790 out of every 1,000,000 men. In an Army of 4,000,000 men, that's a shortage of 139,160 automotive mechanics. Taking only those specialties in which the Army has found major shortages, we find a total of 62,853 lacking in every 300,000 men inducted. That adds up to 838,040 in an Army of 4,000,000 men.

Yes, these shortages of trained man-power—of men trained in the fundamentals of jobs that must be done in a modern army—are serious, much too serious. The situation is fast getting worse. The specialist field is being combed and recombed. The supply of trained men is dwindling by the day. Add to this the shocking fact that more than 200,000 men in this nation already have been deferred from induction into the armed forces because of educational deficiencies—because of illiteracy. These 200,000 men might constitute fifteen combat divisions, yet they must be taught to read and write before they can be utilized by your Army.

What has your Army done to meet and to overcome this situation? It was a simple matter when your Army was small. When we needed specialists, we trained them in Army schools, with Army instructors. There was no premium on instructors. Facilities were ample.

*** An address by Lieutenant General Brehon B. Somervell, commanding general, services of supply, War Department, before American Institute on Education and the War, American University, Washington, D. C., Friday, August 28, 1942.**

When your Army started to expand through the Selective Service, we expanded these facilities. And with the expansion and the adaptation of your Army to modern mechanized warfare, the need for men with basic specialized training increased. We increased our training facilities. We enlarged our staffs of instructors. After Pearl Harbor, with the Army rapidly increasing in size, Army facilities were not enough. We contracted for and secured facilities outside the Army, still using Army instructors. But this was not enough. The need for basic specialists continued to increase. We went further outside the Army. We inducted men who had no basic specialist training but who through aptitude tests showed that they could be developed into specialists. We sent them to civilian schools where they were taught by civilian instructors. They returned to the Army ready to apply their knowledge to its combat counterpart. Even this is not enough. It's not enough because your Army is expanding at a rapid rate. It's not enough because we no longer can create new facilities for training of a non-combatant nature. It's not enough because despite all we've done, shortgages of basically trained men whom we must have are mounting day by day. And it's not enough because we're in a hurry today as never before in our nation's history.

We're in a hurry to put into the field a fighting force capable of overcoming those who seek to destroy everything for which America stands; who seek to destroy America herself. Our job is to teach men to fight. We cannot lose sight of this. We cannot long continue to take the time and facilities needed for this job and use them on a job which could have been done before the induction of men into the Army.

This is your job in this total war. It is the job of the schools and colleges of America to provide the opportunity for every youth to equip himself for a place in winning the war. You must do this, regardless of cost, time, inconvenience, the temporary sidetracking of non-war objectives, or even the temporary scrapping of peace-time courses. *The schools and colleges of America must become pre-induction training centers for our armed forces, leaving the armed forces free to train men in the combat application of the training that you give.*

We cannot win this war unless we mobilize the entire nation. We cannot win this war unless every man, woman and child alters his way of living and finds new ways to put his talent and abilities, his body and mind, at the disposal of the nation. We cannot win this war unless we train every American to make his maximum contribution.

There must be an all-out effort on the education front. Let us be realistic. Every able-bodied boy is destined at the appointed age for the armed services. The tempo of war is such that a complex college education is impossible. Those able to go to college must devote this time to training for the specialized work which the services demand. Those who do not or cannot go to college must begin now, whether they're in school or out of school, to prepare themselves for the tasks which are for them inevitable and unavoidable.

Is this necessary? It is so necessary that all other values depend upon it. I'm passing on to you, to the educators of America, these lines that every person in the Army responsible for the training of men constantly keeps before him: "Be sure that no American soldier is killed or injured because you failed to do your part to provide adequate training." No school in America can fail to do its part. There is no compromise in war. You either lose it or win it. America is in it to win. It may be that you can devote only one or two periods a day to the form of pre-induction training your Army needs. It may be that you will be called upon to devote your entire plant and facilities for the duration of the war. It may be that you can provide special training of particular students. It may be that colleges will have to open their doors to the training of men below

the college level. But you cannot fail. Your cannot fail when the lives of American soldiers depend upon you; when victory itself depends upon you. *If problems of school administration arise, you must find ways of solving them. If more teachers are needed, you must find them. If your teachers need in-service training, the schools and colleges must furnish the training. This is part of an oll-out effort on the education front.*

Great Britain has adopted pre-induction training as a part of its war program. Russia and Canada have done it. And yes, Germany, Japan and Italy have done it. America can do it. America must do it—now. Professionally trained manpower is needed by your Army today as never before in our nation's history. Supplying this need is the way in which the colleges of America can take part in all-out effort on the education front. The schools and colleges of America must see to it that every boy and girl has been given specific education for military or civilian participation in the war effort. They must build up in youth a real understanding of the issues involved in this war. Guidance must be given youth in the physical, mental and other requirements for various specializations in the armed forces so that every individual can do that for which he is best fitted. *The schools of America must provide an education—basic reading, writing and arithmetic—for those 200,000 men who because of illiteracy are unable to take up arms for their country. Your Army has neither the time nor the facilities to teach these three R's. It is you, not the Army, who must do the job for which your experience is best fitted.* The schools and colleges of America must help to provide instruction for civilians, both men and women, who will not be called upon to wear the uniform, but whose specialized training can be utilized by the government. These are the civilian employees recruited by the Civil Service Commission for the War Department in selected occupations. The schools and colleges of America must help provide technical training which may be utilized in the War effort by men and women now employed in non-essential industry.

How are we going to carry out this program? *The War Department is supplying the United States Office of Education with lists of its needs by different categories of specialties. We're telling the Office of Education that we need men with knowledge of physics, mathematics, the fundamentals of radio, principles of mechanics, electrical shop work, automotive mechanics and all the other basic specialized training essential for development of an efficient Army.* But more than just stating our needs, we're preparing outlines and making available technical manuals that will help you in filling these needs. Based on War Department technical manuals the course includes background knowledge and skill which contribute toward twenty-nine Army occupations. All extraneous material is eliminated. Teaching aids are suggested, references are listed for pupil and teacher, demonstrations and laboratory exercises are listed, visual aids are recommended and problems and projects are given. The material covered by the outline takes into consideration the personnel and equipment problems of the schools. Bear in mind that this is a ninety-hour course. This is ninety hours you have saved your Army. Multiply that by the thousands of men we need with such training. Multiply it by other courses you can give. Surely you will make certain now that no American soldier is ever killed or injured because you failed to do your part to provide adequate training. Find out what these needs are from the United States Office of Education. Go back to your schools and colleges. Determine what facilities you have or can set up to meet these needs. Upon request, submit your findings to the United States Office of Education. That office will tell you how to use those facilities and what aids it has available to help you in their use. It's up to you, to the schools and colleges of America, to use every means at your dis-

posal to train men and women for direct participation in the war effort.

I want you to tell Bill Jones that when he has successfully completed a pre-induction course he'll be given a certificate. I want you to tell Bill Jones that that certificate, presented at an Army reception center, will enable him to do any Army job for which he is equipped and a job that the Army must get done.

Let's call these pre-induction training certificates, "Victory Certificates." For Bill Jones, and the thousands of other Bill Jones' you will turn over to the Army, will put us on the road to VICTORY.

◆ THE ART OF TEACHING ◆

Vocabulary in Plane Geometry

By Earl R. Keesler
Hastings-on-Hudson, N. Y.

Since the technical vocabulary of geometry is composed largely of derivatives from Latin and Greek words, the teaching of their history together with pronunciation, meaning, spelling and use should be included as an important part of the course. English equivalents make it much easier to remember definitions which otherwise may be memorized mechanically. For this reason it is sound practice to discuss derivations whenever they may aid in the assimilation of new words. The Latin and history departments are always more than willing to cooperate in such an integration project, while the increased interest and understanding are well worth the extra effort and planning.

For example, consider the name *Plane Geometry* itself. *Plane* stems from the Latin word *planus* meaning level or flat. *Geometry* is formed by a combination of two Greek words, *ge* meaning *earth* with *metrein* meaning *to measure*. It "is probably a translation* of an Egyptian term" denoting surveying, "since that science was well developed in Egypt before the Greeks founded Alexandria." At this point it is interesting for a class to note such other earth sciences as geology, geodesy and geography. Also, from *metrein* we have such related words as *meter*, *metric*, *speedometer* and *chronometer*.

Here is a fine opportunity for optional research assignments. The essential tools are readily available—a dictionary, an encyclopedia and a history of mathematics. Pupils enjoy changes from the usual geometry exercises and are always pleased to find common elements in the subject matter of their various classes.

The following list is suggestive of what can be done:

axiom	—Gr. axioma from axioun, to think worthy
bisect	—L. bi, two with secare, to cut
circumference	—L. circum, around with ferre, to bear or carry
circumscribe	—L. circum with scribere, to write or draw
concurrent	—L. con, together with currere, to run
converse	—L. con with vertere, to turn
diameter	—Gr. dia, through with metrein, to measure
equilateral	—L. aequus, equal with latus, side
geometry	—Gr. ge, earth with metrein, to measure
hypothesis	—Gr. hypo, under with tithenai, to put
inscribe	—L. in, within with scribere, to write or draw
intercept	—L. inter, between with capere, to take
intersect	—L. inter with secare, to cut
isosceles	—Gr. isos, equal with skelos, leg
locus	—L. place
pentagon	—Gr. pente, five with gonia, angle
perimeter	—Gr. peri, around with metrein, to measure
plane	—L. planus, level or flat
polygon	—Gr. polys, many with gonia, angle
postulate	—L. postulare, to demand
quadrant	—L. quadrans-antis, a fourth part
quadrilateral	—L. quattuor, four with latus, side
secant	—L. secare, to cut
subtend	—L. sub, under with tendere, to stretch
tangent	—L. tangere, to touch
transversal	—L. trans, across with vertere, to turn
triangle	—L. tri, three with angulus, angle
trigonometry	—Gr. trigonon, triangle with metrein, to measure

* For further discussion see David Eugene Smith, *History of Mathematics*, Volume II, page 273.

EDITORIAL

Mathematics Essential for the War Effort

THE PLAN to induct boys of eighteen years of age into the armed forces of the United States will doubtless affect very greatly the type of mathematics program to be offered in the secondary schools. It is obvious that the kind of mathematics the boys will need to know to enter the various types of service has not been taught to all of them in late years. In fact only a small percentage of them have taken the necessary amount. The schools must, therefore, be reorganized with these needs in mind. Already the City of New York has regulations governing the intensive program of *war courses*, heretofore generally called *pre-induction courses.* According to the New York Sun of October 27, 1942, the regulations of New York City were classified by Associate Superintendent Frederic Ernst, Chief of the high school division, following conferences with his staff, as follows:

The instructions, contained in a circular sent to high school principals, will serve as a guide to the principals now planning the February semester. One or more of the war courses will be required of every pupil—girls as well as boys—who enter the fifth term or a higher grade of the secondary school division.

Moreover, boys who within one or two terms will become 18 years old are to receive "as much basic technical training as possible." Required courses in their cases therefore will be limited to health education, American history and English. To these may be added "as many of the war courses as possible, irrespective of any group sequences that would otherwise be required for graduation."

In no case, however, will the number of courses taken in any one term be "fewer than the pupils would ordinarily take." The war program will not be permitted to develop into an easy road to graduation.

To Train Teachers

Dr. Ernst declared that although much is now being done to train teachers for the new subject fields, "more will have to be done."

"Some of these war courses are not covered by a specific license, many of them relate to a narrow segment of a broad license field," he wrote. "Therefore, teachers who have prepared themselves or will prepare themselves to teach specific war courses may be permitted to conduct them, up to fifty per cent of their program time, if, in the opinion of the principal, they have the requisite competence and the approval of the division has been secured for such assignment.

"Inevitably, the introduction of an entire war program is due to a change in objective," Dr. Ernst explained. "Naturally, the addition of so many new courses to the curriculum, the substituion of new courses for traditional courses and the redirection of standard courses can only be accomplished by numerous advisory and administrative changes.

Staff Consulted

"The regulations in this new circular were devised by the high school division in conference with a number of high school principals as a means of aiding the schools in the formulation of their war programs for next term. It should be remembered above all that a program of this kind will depend upon the success with which we train teachers to become competent in the new fields."

The circular specifies thirteen approved war courses that may be offered toward graduation, and also lists a group of standard courses that will be accepted as "authorized substitutions" for the war courses. Dr. Ernst informed the principals, however, that the approved war courses should be given wherever possible. "May I strongly emphasize," he stated, "that these substitutions should be made only when necessary."

Courses Listed

The approved war courses listed in the circular follow:

a. Aeronautics III—Prerequisite: Fundamentals of machines.
b. Aeronautics IV.
Note 1—These are advanced courses and, in general, only superior pupils who can also pass a rigorous physical examination should be permitted to register for aeronautics III and IV. In general, these courses should be given in terms 7 and 8.
Note 2—The current courses in aeronautics I and aeronautics II are not to be offered in February, 1943, since the requirements of related mathematics will replace aeronautics I and the course in fundamentals of machines (see below) will replace aeronautics II.
c. Fundamentals of machines.
d. Fundamentals of electricity.
Note 1—These two courses, fundamentals of machines and fundamentals of electricity, should replace the present courses in physics I and II and applied physics I and II. In case of students who require physics for college

entrance, these courses should be listed as physics I and II on college applications.

No. 2—In case of students who desire to take New York State Regents examinations in physics, supplementary work in the areas of light and sound may be given in class or after school to qualify these students.

e. Elementary shop work. To count as one war course.

f. Fundamentals of automotive mechanics; two terms to count as two war courses. Fundamentals of machines (or aero II) and elementary shopwork are prerequisites.

g. Meteorology. To count as one war course.

h. Topography and map reading. To count as one war course.

i. Mechanical drawing and blue print reading. To count as one war course.

j. Fundamentals of radio (two terms, I and II). To count as two war courses. Prerequisite: fundamentals of electricity.

k. Hygiene and home nursing. To count as one war course.

l. Nutrition. To count as one war course. This course may not be offered to students who have had a year of advanced biology or a year of home economics (foods).

m. Radio code practice. To count as one war course.

Substitutions Permitted

Only certain courses may be substituted for war courses and only to the extent indicated according to the circular. They are:

a. Advanced courses in shop, mechanical drawing, electricity and other subjects directly related to the war effort may be substituted for the above list of war courses provided the syllabus for the course has been approved by Dr. Horton. A pupil may offer these approved advanced courses as war courses, term for term, to a maximum of two courses.

b. Girls may offer chemistry I and II, or applied chemistry I and II, term for term, if they are certified to be preparing for nursing.

c. Girls may offer advanced biology I and II, as one war course.

d. Girls with ability to achieve marketable competence may continue stenography and accounting. Girls in this category may substitute stenography 3, or higher, or accounting 3, or higher, for war courses. No more than two war courses (one unit) can be earned in this way by any one pupil.

e. Pupils may offer any two of the following courses (intermediate algebra, trigonometry, solid geometry) as war courses for a maximum of one unit.

f. Girls may offer home economics (foods) I and II, as one war course. Note: Either c or f may be used, but not both.

The circular announces that the term "war courses" will be substituted for "pre-induction courses" and other terms heretofore used to denote the new type of instruction.

The following rules are then prescribed:

A war course pursued for one term, five periods per week, shall carry one-half unit of credit.

Pupils entering the fifth term in February, 1943, and in succeeding terms, shall offer a minimum of four war courses (two units) toward graduation; those entering the sixth term a minimum of three courses (one and a half units) those entering the seventh term a minimum of two courses (one unit) and those entering the eighth term a minimum of one course (one-half unit).

It seems advisable that boys who will reach their eighteenth birthday in one or two terms should receive as much basic technical training as possible. Therefore, they need be required to take only health education, American history and English in addition to as many of the war courses as possible, irrespective of any normal group sequences that would otherwise be required for graduation. In no case, however, should the number of subjects taken in any one term be fewer than the pupil would ordinarily take.

Each war course may be used as one-half unit toward the units required for graduation.

THE STEP already taken by New York City will no doubt be followed by similar action elsewhere. This will result in certain obvious results:

1. Secondary schools will be preparing boys and girls for the armed and allied services.
2. The courses will be offered for *boys* and *girls* and mathematics will be required in all of them.
3. Traditional mathematics, in the sense that overemphasis is placed upon formalism and deductive thinking, will be greatly modified if not eliminated. This means that important applications of algebra and geometry should be introduced. Competence in arithmetic *must* be developed. It looks as if all extra mural requirements are going out for the duration if not for good.
4. Pupils who have not already had a sufficient amount of mathematics to date will be given a course in Basic Related Mathematics to be taken parallel with the required war courses (Four in New York City).
5. Teachers from other fields will have to be trained to teach some of the expanded mathematics program. This is a place where great care should be taken to make sure that teachers are properly equipped to do satisfactory work.
6. Textbooks are now being written to meet present needs and they should soon be available.
7. Whatever gains can be made in the reorganization of materials for the war effort should be conserved for the peace that follows.

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IN OTHER PERIODICALS

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The American Mathematical Monthly
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